Mutual Impedance Transformer Model

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Rev 6

This document describes a new transformer model that models the self and mutual impedances of a transformer. This allows the model to account for the fact that the losses in a winding due to current in that winding are affected by the currents in the other windings. The model accomplishes this by selecting the model coefficients that determine the real parts of the self and mutual impedances with a least-squares curve fitting routing which matches the measured and modeled self and mutual resistances given a set of self and mutual inductances that were measured at a low frequency where the losses are small.

Transformer construction details:

Winding 1: 64 turns of #19 wire in two layers with one layer of 3M #44 tape between the layers.

Two layers of 3M #44 tape between the primary and secondary windings.

Winding 2: 94 turns of #23 wire in two layers with one layer of 3M #44 tape between the layers.

All winding layers were wound in the same direction. The two layers in each of the two windings were connected in series with a jumper. This winding method was selected to raise the self-resonant frequencies. The wire sizes were chosen to ensure that ac losses would be high at relatively low frequencies before capacitive effects become significant.

Core: ETD49 3C85 with a 122 mil gap in the center leg.

Set the array origin equal to 1. $\quad ORIGIN := 1$

The self impedances and leakage impedances were measured at 13 frequencies.

Define a counter variable for the frequencies. $\quad n := 1 .. 13$

Enter the frequencies at which the measurements were made.

$$f := \begin{pmatrix} 1.0000\cdot 10^{3} \\ 2.0178\cdot 10^{3} \\ 4.0180\cdot 10^{3} \\ 8.0010\cdot 10^{3} \\ 11.9050\cdot 10^{3} \\ 16.1450\cdot 10^{3} \\ 20.2220\cdot 10^{3} \\ 40.2690\cdot 10^{3} \\ 60.7150\cdot 10^{3} \\ 80.1870\cdot 10^{3} \\ 120.9010\cdot 10^{3} \\ 161.8030\cdot 10^{3} \\ 200.0000\cdot 10^{3} \end{pmatrix}$$

Compute the corresponding radian frequencies.

$$\omega := 2\cdot\pi\cdot f$$

Enter the series resistance and inductance values at the selected frequencies for winding 1.

$$R_{11} := \begin{pmatrix} 0.124150 \\ 0.138823 \\ 0.198123 \\ 0.450103 \\ 0.833485 \\ 1.391000 \\ 2.025170 \\ 5.939820 \\ 9.978400 \\ 13.418600 \\ 19.831100 \\ 25.999800 \\ 32.689900 \end{pmatrix}$$

$$L_{11} := \begin{pmatrix} 536.71\cdot10^{-6} \\ 536.79\cdot10^{-6} \\ 536.77\cdot10^{-6} \\ 535.90\cdot10^{-6} \\ 534.85\cdot10^{-6} \\ 533.49\cdot10^{-6} \\ 531.91\cdot10^{-6} \\ 523.68\cdot10^{-6} \\ 517.75\cdot10^{-6} \\ 514.98\cdot10^{-6} \\ 517.16\cdot10^{-6} \\ 527.53\cdot10^{-6} \\ 543.67\cdot10^{-6} \end{pmatrix}$$

Enter the series resistance and inductance values at the selected frequencies for winding 2.

$$R_{22} := \begin{pmatrix} 0.535511 \\ 0.575817 \\ 0.748045 \\ 1.438690 \\ 2.491440 \\ 4.016490 \\ 5.753720 \\ 16.004700 \\ 26.129100 \\ 34.617500 \\ 50.164400 \\ 64.942100 \\ 80.615400 \end{pmatrix}$$

$$L_{22} := \begin{pmatrix} 1.202670 \cdot 10^{-3} \\ 1.202660 \cdot 10^{-3} \\ 1.202310 \cdot 10^{-3} \\ 1.200230 \cdot 10^{-3} \\ 1.197110 \cdot 10^{-3} \\ 1.193060 \cdot 10^{-3} \\ 1.188550 \cdot 10^{-3} \\ 1.164460 \cdot 10^{-3} \\ 1.147610 \cdot 10^{-3} \\ 1.139130 \cdot 10^{-3} \\ 1.140810 \cdot 10^{-3} \\ 1.169150 \cdot 10^{-3} \\ 1.196300 \cdot 10^{-3} \end{pmatrix}$$

Enter the leakage series resistance and inductance values at the selected frequencies for winding 1 with winding 2 shorted.

$$R_{leak\_12} := \begin{pmatrix} 0.345721 \\ 0.348931 \\ 0.357603 \\ 0.392190 \\ 0.444715 \\ 0.518145 \\ 0.600019 \\ 1.064930 \\ 1.517580 \\ 1.894580 \\ 2.533690 \\ 3.035440 \\ 3.418430 \end{pmatrix} \qquad L_{leak\_12} := \begin{pmatrix} 18.2881\cdot 10^{-6} \\ 16.4255\cdot 10^{-6} \\ 15.9208\cdot 10^{-6} \\ 15.6879\cdot 10^{-6} \\ 15.4870\cdot 10^{-6} \\ 15.2313\cdot 10^{-6} \\ 14.9721\cdot 10^{-6} \\ 13.6671\cdot 10^{-6} \\ 12.6158\cdot 10^{-6} \\ 11.8945\cdot 10^{-6} \\ 10.8970\cdot 10^{-6} \\ 10.2826\cdot 10^{-6} \\ 9.89829\cdot 10^{-6} \end{pmatrix}$$

Enter the leakage series resistance and inductance values at the selected frequencies for winding 2 with winding 1 shorted.

$$R_{leak\_21} := \begin{pmatrix} 0.784471 \\ 0.790075 \\ 0.810430 \\ 0.890508 \\ 1.012290 \\ 1.183050 \\ 1.374010 \\ 2.443350 \\ 3.460090 \\ 4.301800 \\ 5.572630 \\ 6.848580 \\ 7.698120 \end{pmatrix} \qquad L_{leak\_21} := \begin{pmatrix} 36.8235 \cdot 10^{-6} \\ 35.6899 \cdot 10^{-6} \\ 35.3312 \cdot 10^{-6} \\ 34.9915 \cdot 10^{-6} \\ 34.5612 \cdot 10^{-6} \\ 33.9943 \cdot 10^{-6} \\ 33.3869 \cdot 10^{-6} \\ 30.3168 \cdot 10^{-6} \\ 27.8997 \cdot 10^{-6} \\ 26.2546 \cdot 10^{-6} \\ 23.9973 \cdot 10^{-6} \\ 22.6304 \cdot 10^{-6} \\ 21.7782 \cdot 10^{-6} \end{pmatrix}$$

Enter the dc resistance of winding 1. $R_{dc1} := 0.119$

Enter the dc resistance of winding 2. $R_{dc2} := 0.521$

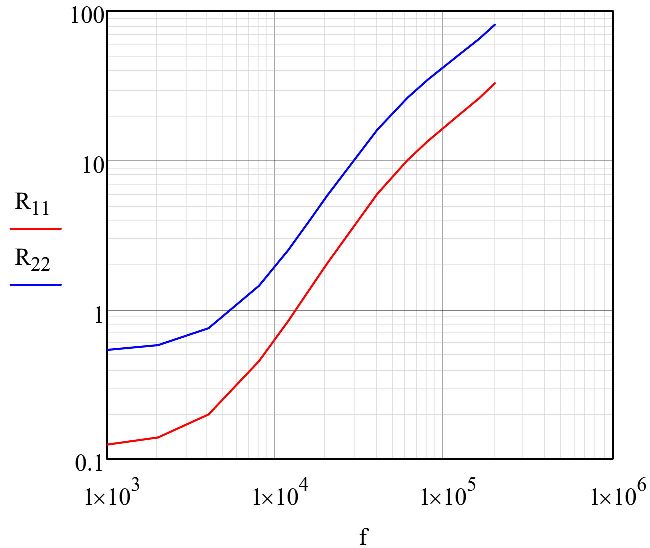

Fig. 1. Winding resistances.

The winding resistance curves have the expected shape. They start with a dc value to which an ac resistance is added. The ac resistance is proportional the the frequency squared at low frequencies where the proximity effect is dominant. The ac resistance is proportional to the frequency at higher frequencies, where the skin effect is dominant. The ac resistance of a winding is obtained by subtracting the dc resistance from the measured resistance. Figs. 2 and 3 show ac resistance plots with asymptotes for the proximity effect and skin effect loss regimes.

Compute the ac resistances.

$R_{11ac} := R_{11} - R_{dc1}$ $R_{22ac} := R_{22} - R_{dc2}$

Extract the coefficients for the proximity effect asymptotes functions.

Define frequency and resistance vectors for the proximity effect regime.

$m := 1..7$

$fp_m := f_m$ $\quad R_{11acp_m} := R_{11ac_m}$ $\quad R_{22acp_m} := R_{22ac_m}$

Define a frequency squared function. $\quad F(f) := f^2$

Compute the coefficients for the proximity effect asymptotes.

$PE1 := \mathrm{linfit}(fp, R_{11acp}, F)$ $\quad PE2 := \mathrm{linfit}(fp, R_{22acp}, F)$

$PE1 = 4.7563 \times 10^{-9}$ $\quad PE2 = 1.3068 \times 10^{-8}$

Compute the proximity effect asymptote vectors for plotting.

$Prox1_n := PE1 \cdot (f_n)^2$ $\quad Prox2_n := PE2 \cdot (f_n)^2$

Define frequency and resistance vectors for the skin effect regime.

$m := 1..4$

$fs_m := f_{m+9}$ $\quad R_{11acs_m} := R_{11ac_{m+9}}$ $\quad R_{22acs_m} := R_{22ac_{m+9}}$

Define a frequency function. $\quad F(f) := f$

Compute the coefficients for the skin effect asymptotes.

$SE1 := \mathrm{linfit}\left(fs, R_{11acs}, F\right)$ $\qquad$ $SE2 := \mathrm{linfit}\left(fs, R_{22acs}, F\right)$

$SE1 = 1.6224 \times 10^{-4}$ $\qquad$ $SE2 = 4.0330 \times 10^{-4}$

Compute the skin effect asymptote vectors for plotting.

$Skin1_n := SE1 \cdot f_n$ $\qquad$ $Skin2_n := SE2 \cdot f_n$

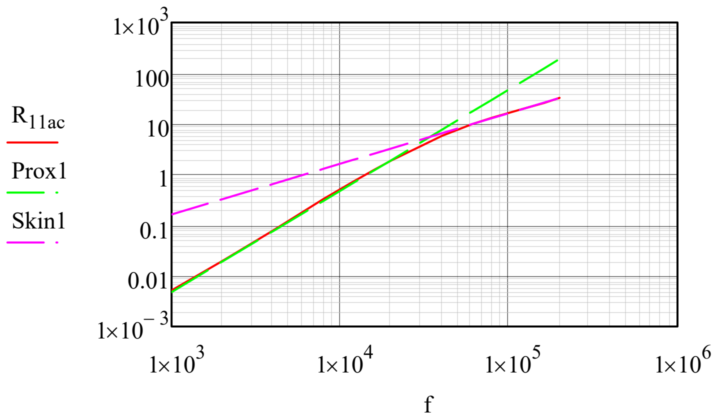

Fig. 2. Proximity effect and skin effect asymptotes for winding 1.

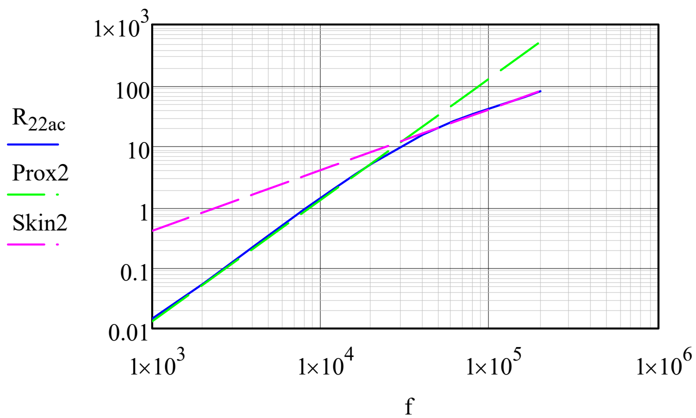

Fig. 3. Proximity effect and skin effect asymptotes for winding 2.

Plot the self inductances of the windings.

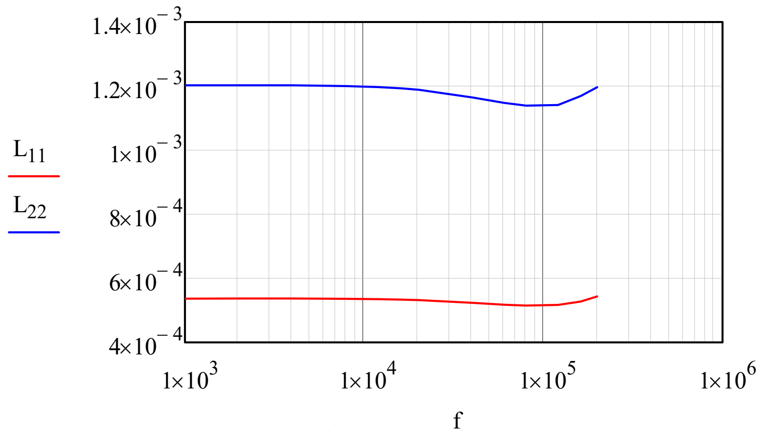

Fig. 4. Winding Inductances.

Ideally, the winding inductances would drop monotonically. The winding capacitances makes the inductance values start to rise again as resonance is approached.

Plot the leakage resistances and inductances.

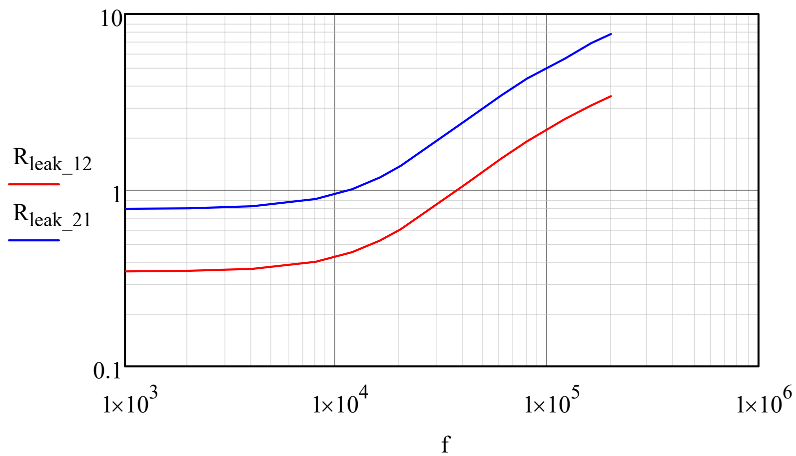

Fig. 5. Leakage resistances.

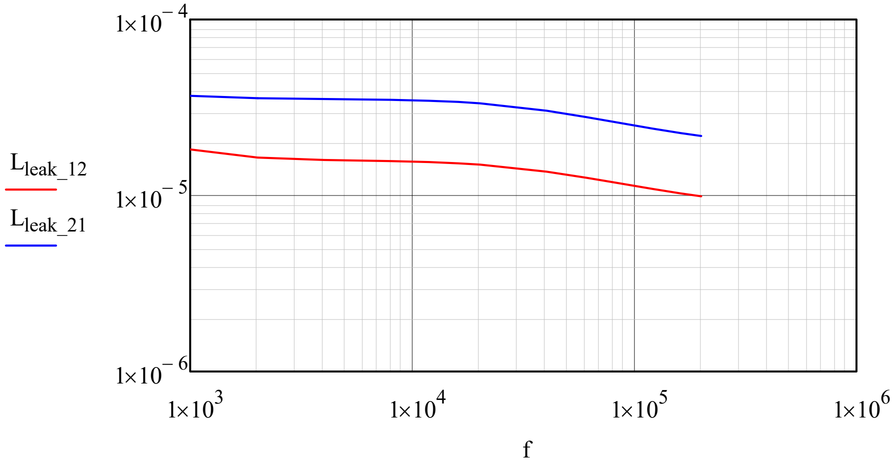

Fig. 6. Leakage inductances.

We begin the process of modeling the transformer by defining the voltages and currents as shown in Fig. 7.

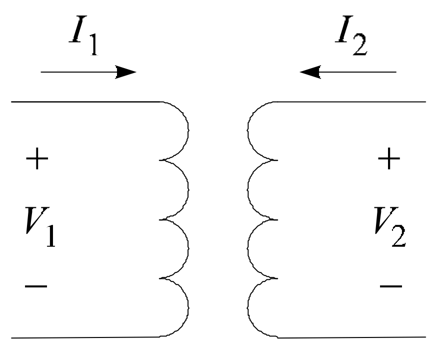

Fig. 7. Transformer voltage and currents

As with any two-port network, the transformer voltages and currents can be described in the frequency domain in terms of self and mutual impedances.

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 \quad (1)$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2 \quad (2)$$

The self impedances can be expressed in terms of the winding resistances and inductances.

$$Z_{mm} = R_{mm} + j \cdot \omega \cdot L_{mm} \quad . \quad (3)$$

Compute the self impedances from the measured resistance and inductance values.

$$Z_{11_n} := R_{11_n} + j \cdot \omega_n \cdot L_{11_n} \qquad Z_{22_n} := R_{22_n} + j \cdot \omega_n \cdot L_{22_n}$$

The leakage impedances can be expressed in terms of the leakage resistances and inductances.

$$Z_{leak\_mn} = R_{leak\_mn} + j \cdot \omega \cdot L_{leak\_mn} \quad (4)$$

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Compute the leakage impedances from the measured leakage resistance and inductance values.

$$Z_{leak\_12_n} := R_{leak\_12_n} + j\cdot\omega_n\cdot L_{leak\_12_n} \qquad Z_{leak\_21_n} := R_{leak\_21_n} + j\cdot\omega_n\cdot L_{leak\_21_n}$$

The mutual impedances can be expressed in terms of the self and leakage impedances. The leakage impedance of winding m with respect to winding n can be found by applying a sinusoidal signal to winding m while winding n is shorted. V_m and I_m are phasors.

$$Z_{leak\_mn} = \frac{V_m}{I_m} \tag{5}$$

$$V_n = 0 \tag{6}$$

Equations (1) and (2) can be expressed in terms of m and n. A fundamental principle of network theory is that $Z_{mn} = Z_{nm}$. Thus only Z_{mn} will be used in the following equations.

$$V_m = Z_{mm}\cdot I_m + Z_{mn}\cdot I_n \tag{7}$$

$$V_n = Z_{mn}\cdot I_m + Z_{nn}\cdot I_n \tag{8}$$

Solve (8) for I_n given (6).

$$I_n = \frac{-Z_{mn}\cdot I_1}{Z_{nn}} \tag{9}$$

Substitute (9) into (7).

$$V_m = Z_{mm}\cdot I_m + Z_{mn}\cdot\frac{-Z_{mn}\cdot I_m}{Z_{nn}} \tag{10}$$

Divide both sides of (10) by I_m so that the left side is equal to the right side of (5).

$$\frac{V_m}{I_m} = Z_{mm} - \frac{{Z_{mn}}^2}{Z_{nn}} \tag{11}$$

Substitute (11) into (5) to obtain an equation for the leakage impedance.

$$Z_{leak\_mn} = Z_{mm} - \frac{{Z_{mn}}^2}{Z_{nn}} \tag{12}$$

Solve (12) for Z_{mn} to obtain an expression for the mutual impedance. The principal square root is used.

$$Z_{mn} = \sqrt{\left(Z_{mm} - Z_{leak\_mn}\right) \cdot Z_{nn}} \tag{13}$$

Compute the mutual impedances.

$$Z_{12_n} := \sqrt{\left(Z_{11_n} - Z_{leak\_12_n}\right) \cdot Z_{22_n}} \qquad Z_{21_n} := \sqrt{\left(Z_{22_n} - Z_{leak\_21_n}\right) \cdot Z_{11_n}}$$

Compute the mutual resistances and inductances.

$$R_{12_n} := \mathrm{Re}\left(Z_{12_n}\right) \qquad R_{21_n} := \mathrm{Re}\left(Z_{21_n}\right) \qquad L_{12_n} := \frac{\mathrm{Im}\left(Z_{12_n}\right)}{\omega_n} \qquad L_{21_n} := \frac{\mathrm{Im}\left(Z_{21_n}\right)}{\omega_n}$$

Since we expect Z_{mn} to equal Z_{nm}, we would expect R_{12} to equal R_{21}, and L_{12} to equal L_{21}. This is demonstrated in Figs. 8 and 9.

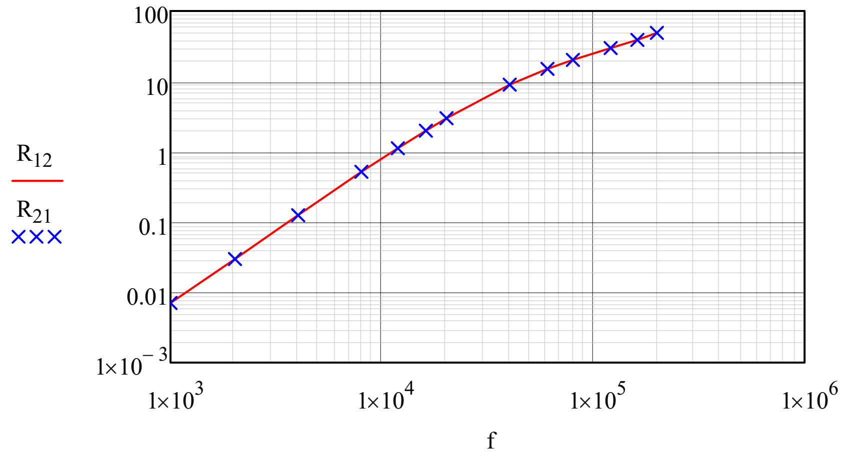

Fig. 8. Mutual resistances.

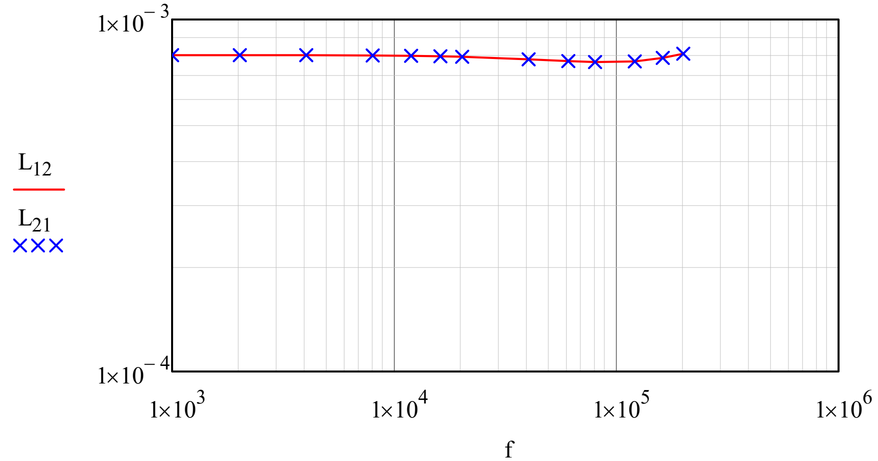

Fig. 9. Mutual inductances.

We can demonstrate that the computed mutual impedance values are correct by using them to compute the leakage resistances and inductances. The computed values can then be compared with the measured values.

Compute the transformer admittance matrix at each frequency.

$$Y_n := \begin{pmatrix} Z_{11_n} & Z_{12_n} \\ Z_{12_n} & Z_{22_n} \end{pmatrix}^{-1}$$

Compute the winding currents at each frequency when 1 volt is applied to winding m, and winding n is shorted.

$$I_{leak\_12_n} := Y_n \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad I_{leak\_21_n} := Y_n \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Compute the leakage impedances at each frequency.

$$Z_{leak\_12c_n} := \frac{1}{\left(I_{leak\_12_n}\right)_1} \qquad Z_{leak\_21c_n} := \frac{1}{\left(I_{leak\_21_n}\right)_2}$$

Compute the leakage resistances at each frequency.

$$R_{leak\_12c_n} := \mathrm{Re}\left(Z_{leak\_12c_n}\right) \qquad R_{leak\_21c_n} := \mathrm{Re}\left(Z_{leak\_21c_n}\right)$$

Compute the leakage inductances at each frequency.

$$L_{leak\_12c_n} := \frac{\mathrm{Im}\left(Z_{leak\_12c_n}\right)}{\omega_n} \qquad L_{leak\_21c_n} := \frac{\mathrm{Im}\left(Z_{leak\_21c_n}\right)}{\omega_n}$$

Plot the measured and computed leakage resistances and inductances.

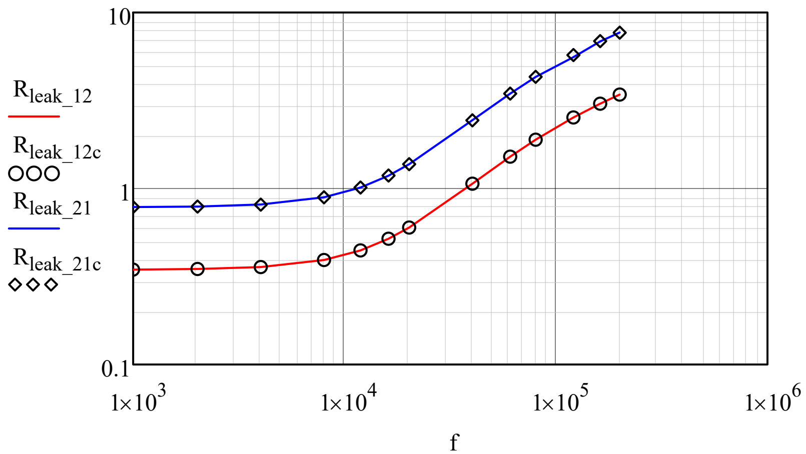

Fig. 10. Measured and computed leakage resistances.

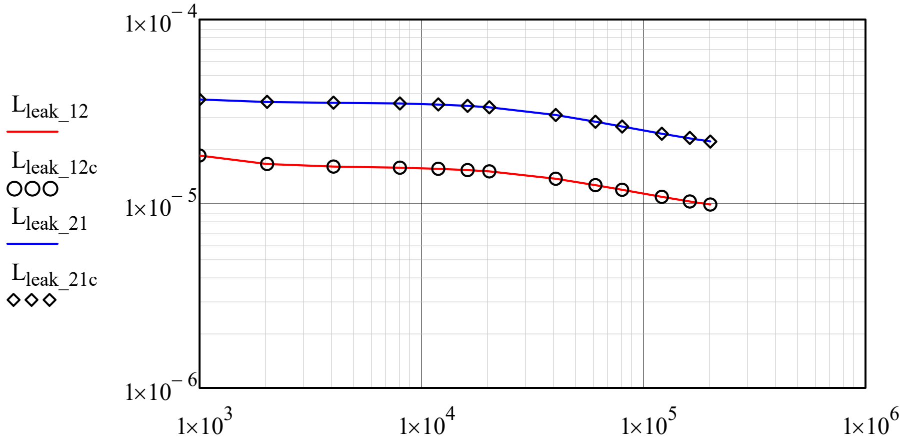

Fig. 11. Measured and computed leakage inductances.

The impedance matrix description of the transformer can be approximated with the circuit model shown below in Fig. 12. For each winding, there is a resistor representing the dc resistance of that winding and a main inductor representing the maximum low-frequency inductance for that winding. The main inductor and the dc resistance for each winding are connected in series between the electrical terminals of that winding. There is also a set of auxiliary circuits for each winding that is shown in a row to the left of the winding. Each auxiliary circuit consists of an auxiliary inductor that is connected in parallel with an auxiliary resistor. The bottom terminals of all of the inductors in each set are connected together to prevent floating nodes, which are not allowed in circuit simulators.

The schematic diagram shows two auxiliary circuits for each winding, but the model could be extended to include more auxiliary circuits. Increasing the number of auxiliary circuits increases the frequency range in which the skin effect can be modeled.

The main inductors are coupled to each other and to each of the auxiliary inductors. The auxiliary inductors are not coupled to each other. It is, of course, impossible to construct a magnetic device in which a set of uncoupled windings are all coupled some other winding. This arrangement is useful as a model, however, and it is possible to describe it mathematically, and to model it in circuit simulators.

The model has one more degree of freedom than is necessary, so the inductances of the auxiliary inductors in each set are assigned a value equal to the inductance of the main winding associated with that set.

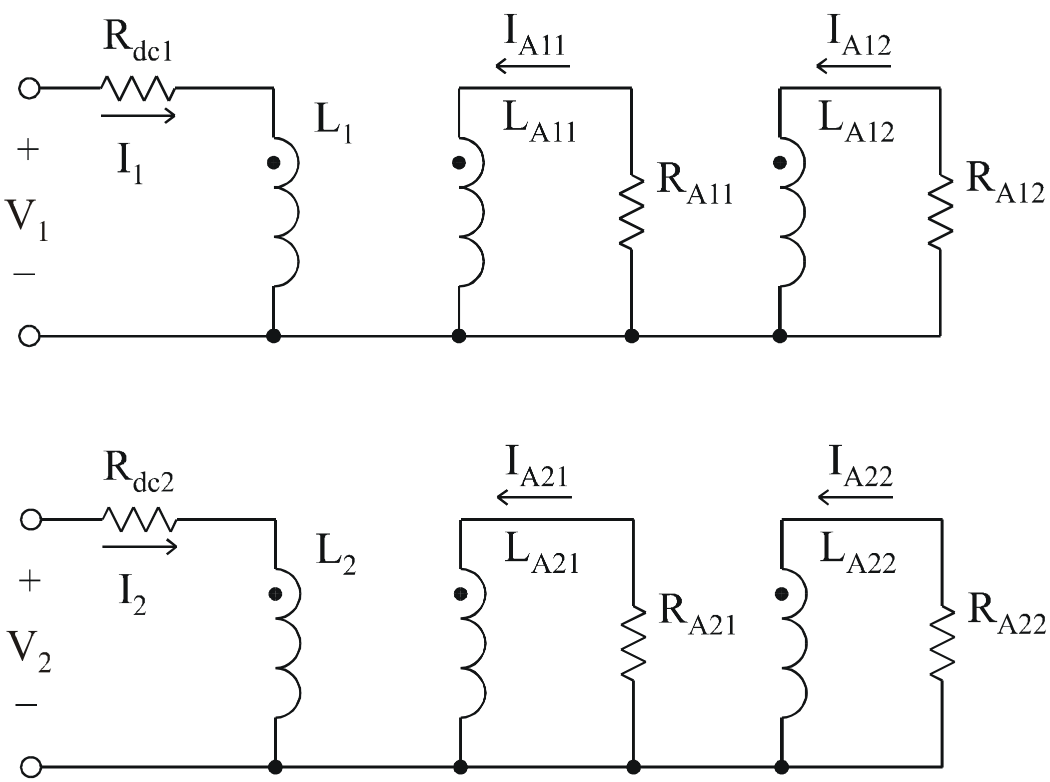

Figure 12. Schematic diagram of transformer circuit model.

We now define several variables and matrices that will be used in an equation that describes the circuit of Fig. 12.

The number of physical windings is designated as N, which is 2 in Fig. 12. The counter variables λ and ν range from 1 to N.

The number of auxiliary circuits for each winding is designated as r, which is 2 in Fig. 12. The counter variable κ ranges from 1 to r.

$N := 2$ $\qquad$ $r := 2$

The mutual inductance between L_1 and L_2 is designated L_{12}.

The mutual inductances between the main inductors and the auxiliary inductors are grouped in a matrix M in which each element $M_{\lambda q}$ corresponds to the mutual inductance between a main inductor L_λ and an auxiliary inductor $L_{A\nu\kappa}$, where $q = (\nu - 1)\cdot r + \kappa$. M has N rows and $N\cdot r$ columns.

$r = 2$	$\nu = 1 \quad \kappa = 1$	$\nu = 1 \quad \kappa = 2$	$\nu = 2 \quad \kappa = 1$	$\nu = 2 \quad \kappa = 2$
	$q = (1 - 1)\cdot 2 + 1 = 1$	$q = (1 - 1)\cdot 2 + 2 = 2$	$q = (2 - 1)\cdot 2 + 1 = 3$	$q = (2 - 1)\cdot 2 + 2 = 4$
$\lambda = 1$	M_{11}	M_{12}	M_{13}	M_{14}
	$L_1 \quad L_{A11}$	$L_1 \quad L_{A12}$	$L_1 \quad L_{A21}$	$L_1 \quad L_{A22}$
$\lambda = 2$	M_{21}	M_{22}	M_{23}	M_{24}
	$L_2 \quad L_{A11}$	$L_2 \quad L_{A12}$	$L_2 \quad L_{A21}$	$L_2 \quad L_{A22}$

Table 1. Auxiliary mutual inductance relationships

Define a matrix that corresponds to the impedances of the mutual inductances.

$$Zm = j\cdot\omega\cdot M \tag{14}$$

The values of the resistors and inductors in the auxiliary circuits can be arranged into two vectors that will be used in subsequent calculations.

$$R_A = \begin{pmatrix} R_{A11} \\ R_{A12} \\ R_{A21} \\ R_{A22} \end{pmatrix} \tag{15}$$

$$L_A = \begin{pmatrix} L_{A11} \\ L_{A12} \\ L_{A21} \\ L_{A22} \end{pmatrix} = \begin{pmatrix} L_1 \\ L_1 \\ L_2 \\ L_2 \end{pmatrix} \tag{16}$$

Define a matrix that describes the impedances resulting from auxiliary inductors and resistors. Za is a diagonal matrix with $r \cdot N$ diagonal entries in which

$$Za_{mm} = R_{A_m} + j \cdot \omega \cdot L_{A_m} \tag{17}$$

Define a matrix of the main self and mutual inductances measured at the lowest frequency.

$$Lb := \begin{pmatrix} L_{11_1} & L_{12_1} \\ L_{12_1} & L_{22_1} \end{pmatrix} \tag{18}$$

Define a matrix that describes the impedances resulting from the dc resistances and the self and mutual impedances of the main inductors.

$$Zb = \begin{pmatrix} R_{dc1} + j \cdot \omega \cdot Lb_{11} & 1j\omega \cdot Lb_{12} \\ 1j\omega \cdot Lb_{12} & R_{dc2} + j \cdot \omega \cdot Lb_{22} \end{pmatrix} \tag{19}$$

The circuit of Fig. 12 can be described in the frequency domain with the following matrix equation:

$$\begin{pmatrix} V_1 \\ V_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = Zsys \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_{A11} \\ I_{A12} \\ I_{A21} \\ I_{A22} \end{pmatrix} \tag{20}$$

where

$$Zsys = \begin{pmatrix} Zb & Zm \\ Zm^T & Za \end{pmatrix} \tag{21}$$

A least-squares curve fitting routine will be used to obtain the values of M and R_A. The routine will match the calculated and measured self and mutual resistances.

The self resistances of the circuit model can be computed with:

$$R_{11} = R_{dc1} + \sum_{\kappa=1}^{2} \left[\frac{\omega^2 \cdot R_{A_\kappa}}{\left(R_{A_\kappa}\right)^2 + \omega^2 \cdot \left(Lb_{1,1}\right)^2} \cdot \left(M_{1,\kappa}\right)^2 \right] + \sum_{\kappa=1}^{2} \left[\frac{\omega^2 \cdot R_{A_{\kappa+r}}}{\left(R_{A_{\kappa+r}}\right)^2 + \omega^2 \cdot \left(Lb_{2,2}\right)^2} \cdot \left(M_{1,\kappa+r}\right)^2 \right] \tag{22}$$

$$R_{22} = R_{dc2} + \sum_{\kappa=1}^{2} \left[\frac{\omega^2 \cdot R_{A_{\kappa+r}}}{\left(R_{A_{\kappa+r}}\right)^2 + \omega^2 \cdot \left(Lb_{2,2}\right)^2} \cdot \left(M_{2,\kappa+r}\right)^2 \right] + \sum_{\kappa=1}^{2} \left[\frac{\omega^2 \cdot R_{A_\kappa}}{\left(R_{A_\kappa}\right)^2 + \omega^2 \cdot \left(Lb_{1,1}\right)^2} \cdot \left(M_{2,\kappa}\right)^2 \right] \tag{23}$$

The self inductances of the circuit model can be computed with:

$$L_{11} = Lb_{1,1} - \sum_{\kappa=1}^{2}\left[\frac{\omega^2\cdot Lb_{1,1}}{\left(R_{A_\kappa}\right)^2 + \omega^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{1,\kappa}\right)^2\right] - \sum_{\kappa=1}^{2}\left[\frac{\omega^2\cdot Lb_{2,2}}{\left(R_{A_{\kappa+r}}\right)^2 + \omega^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{1,\kappa+r}\right)^2\right] \qquad (24)$$

$$L_{22} = Lb_{2,2} - \sum_{\kappa=1}^{2}\left[\frac{\omega^2\cdot Lb_{2,2}}{\left(R_{A_{\kappa+r}}\right)^2 + \omega^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{2,\kappa+r}\right)^2\right] - \sum_{\kappa=1}^{2}\left[\frac{\omega^2\cdot Lb_{1,1}}{\left(R_{A_\kappa}\right)^2 + \omega^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{2,\kappa}\right)^2\right] \qquad (25)$$

I have not formally derived (22-25) but I guessed at the forms by inspection of (28) in reference [1]. I then checked the results numerically with the following method.

Define two reduced-order system matrices $Zsys\_1$ and $Zsys\_2$, where $Zsys\_n$ is obtained by removing all of the first N rows and columns from $Zsys$ except the nth row and column Compute the current in each winding when 1 volt at a selected test frequency is applied to that winding, and the other winding is disconnected.

$$\begin{pmatrix} I_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = Zsys1^{-1}\cdot\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (26)$$

$$\begin{pmatrix} I_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = Zsys2^{-1}\cdot\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (27)$$

The impedance at the test frequency is equal to the reciprocal of the current.

$$Z_{11} = \frac{1}{I_1} \tag{28}$$

$$Z_{22} = \frac{1}{I_2} \tag{29}$$

I have not yet developed formulas similar to (22-25) for the leakage impedances, so I compute them using the system matrix as follows:

To calculate the leakage impedance of winding 1, compute the winding and auxiliary currents at a test frequency when 1 volt is applied to winding 1, and winding 2 is shorted.

$$\mathrm{Ileak12} = \mathrm{Zsys}^{-1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{30}$$

Compute the leakage impedance of winding 1 at the test frequency.

$$\mathrm{Zleak12C} = \frac{1}{\mathrm{Ileak12}_1} \tag{31}$$

Compute the leakage resistance and inductance of winding 1 at the test frequency.

$$\mathrm{Rleak12C} = \mathrm{Re}(\mathrm{Zleak12C}) \tag{32}$$

$$\mathrm{Lleak12C} = \frac{\mathrm{Im}(\mathrm{Zleak12C})}{\omega} \tag{33}$$

The leakage resistance and inductance of winding 2 is calculated in the same manner. The curve-fitting routine, however, only uses the leakage resistance of winding 1.

Since Zb depends only on known values, we can compute an array of matrices Zb for each measured frequency outside of the curve-fitting routine.

$$\mathrm{Zb}_n := \begin{pmatrix} R_{dc1} & 0 \\ 0 & R_{dc2} \end{pmatrix} + j\cdot\omega_n\cdot\mathrm{Lb}$$

Assign values to L_A in accordance with (16).

$$L_A := \begin{pmatrix} \mathrm{Lb}_{1,1} \\ \mathrm{Lb}_{1,1} \\ \mathrm{Lb}_{2,2} \\ \mathrm{Lb}_{2,2} \end{pmatrix} \qquad L_A = \begin{pmatrix} 5.3671 \times 10^{-4} \\ 5.3671 \times 10^{-4} \\ 1.2027 \times 10^{-3} \\ 1.2027 \times 10^{-3} \end{pmatrix}$$

The values of the resistors and the mutual inductances can be obtained with a solve block which minimizes the normalized least-squared error between the calculated and measured winding resistances, and the calculated and measured leakage resistances.

The solve block requires a set of guess values for R_A and M. It should be possible to derive guess values from the proximity effect and skin effect asymptotes computed above. For now, we can use some values that were obtained through trial and error.

Guess values

$$R_A := \begin{pmatrix} 400 \\ 6\cdot 10^3 \\ 350 \\ 6\cdot 10^3 \end{pmatrix} \qquad M := \begin{pmatrix} 1.3\cdot 10^{-4} & 1\cdot 10^{-4} & 1.6\cdot 10^{-4} & 1.6\cdot 10^{-4} \\ 1.6\cdot 10^{-4} & 1.6\cdot 10^{-4} & 3.3\cdot 10^{-4} & 3.2\cdot 10^{-4} \end{pmatrix}$$

Given

Match self resistances.

$$\sum_{n=1}^{13}\left[1-\frac{R_{dc1}+\sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_\kappa}}{\left(R_{A_\kappa}\right)^2+(\omega_n)^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{1,\kappa}\right)^2\right]+\sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_{\kappa+r}}}{\left(R_{A_{\kappa+r}}\right)^2+(\omega_n)^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{1,\kappa+r}\right)^2\right]}{R_{11_n}}\right]^2 = 0$$

$$\sum_{n=1}^{13}\left[1-\frac{R_{dc2}+\sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_{\kappa+r}}}{\left(R_{A_{\kappa+r}}\right)^2+(\omega_n)^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{2,\kappa+r}\right)^2\right]+\sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_\kappa}}{\left(R_{A_\kappa}\right)^2+(\omega_n)^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{2,\kappa}\right)^2\right]}{R_{22_n}}\right]^2 = 0$$

Match leakage resistances.

$$\sum_{n=1}^{13}\left[1-\mathrm{Re}\left[\frac{1}{\left[\begin{bmatrix}(Zb_n)_{1,1} & (Zb_n)_{1,2} & j\cdot\omega_n\cdot M_{1,1} & j\cdot\omega_n\cdot M_{1,2} & j\cdot\omega_n\cdot M_{1,3} & j\cdot\omega_n\cdot M_{1,4}\\ (Zb_n)_{2,1} & (Zb_n)_{2,2} & j\cdot\omega_n\cdot M_{2,1} & j\cdot\omega_n\cdot M_{2,2} & j\cdot\omega_n\cdot M_{2,3} & j\cdot\omega_n\cdot M_{2,4}\\ j\cdot\omega_n\cdot M_{1,1} & j\cdot\omega_n\cdot M_{2,1} & R_{A_1}+j\cdot\omega_n\cdot L_{A_1} & 0 & 0 & 0\\ j\cdot\omega_n\cdot M_{1,2} & j\cdot\omega_n\cdot M_{2,2} & 0 & R_{A_2}+j\cdot\omega_n\cdot L_{A_2} & 0 & 0\\ j\cdot\omega_n\cdot M_{1,3} & j\cdot\omega_n\cdot M_{2,3} & 0 & 0 & R_{A_3}+j\cdot\omega_n\cdot L_{A_3} & 0\\ j\cdot\omega_n\cdot M_{1,4} & j\cdot\omega_n\cdot M_{2,4} & 0 & 0 & 0 & R_{A_4}+j\cdot\omega_n\cdot L_{A_4}\end{bmatrix}^{-1}\cdot\begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}\right]_1}\right]\cdot\frac{1}{R_{leak\_12_n}}\right]^2 = 0$$

Constraints to prevent negative component values

$R_{A_1} > 0$ $\quad M_{1,1} > 0$ $\quad M_{2,1} > 0$

$R_{A_2} > 0$ $\quad M_{1,2} > 0$ $\quad M_{2,2} > 0$

$R_{A_3} > 0$ $\quad M_{1,3} > 0$ $\quad M_{2,3} > 0$

$R_{A_4} > 0$ $\quad M_{1,4} > 0$ $\quad M_{2,4} > 0$

$X := \mathrm{Minerr}\left(R_A, M\right)$

$ERR = 6.0050 \times 10^{-3}$

$X = \begin{pmatrix} \{4,1\} \\ \{2,4\} \end{pmatrix}$ $R_A := X_1$ $M := X_2$

$\begin{pmatrix} R_{A_1} \\ R_{A_2} \\ R_{A_3} \\ R_{A_4} \\ M_{1,1} \\ M_{1,2} \\ M_{1,3} \\ M_{1,4} \\ M_{2,1} \\ M_{2,2} \\ M_{2,3} \\ M_{2,4} \end{pmatrix} =$

	1
1	277.5980
2	$6.7350 \cdot 10^{3}$
3	344.5523
4	$6.2452 \cdot 10^{3}$
5	$1.2658 \cdot 10^{-4}$
6	$1.7846 \cdot 10^{-4}$
7	$1.4899 \cdot 10^{-4}$
8	$1.5551 \cdot 10^{-4}$
9	$1.2946 \cdot 10^{-4}$
10	$3.0500 \cdot 10^{-4}$
11	$3.1596 \cdot 10^{-4}$
12	$3.1620 \cdot 10^{-4}$

We can now compute the self, mutual, and leakage resistances and inductances using the model, and compare them with the measured values.

Compute the self resistances and inductances.

$$R_{11C_n} := R_{dc1} + \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_\kappa}}{\left(R_{A_\kappa}\right)^2 + (\omega_n)^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{1,\kappa}\right)^2\right] + \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_{\kappa+r}}}{\left(R_{A_{\kappa+r}}\right)^2 + (\omega_n)^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{1,\kappa+r}\right)^2\right]$$

$$R_{22C_n} := R_{dc2} + \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_{\kappa+r}}}{\left(R_{A_{\kappa+r}}\right)^2 + (\omega_n)^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{2,\kappa+r}\right)^2\right] + \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot R_{A_\kappa}}{\left(R_{A_\kappa}\right)^2 + (\omega_n)^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{2,\kappa}\right)^2\right]$$

$$L_{11C_n} := Lb_{1,1} - \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot Lb_{1,1}}{\left(R_{A_\kappa}\right)^2 + (\omega_n)^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{1,\kappa}\right)^2\right] - \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot Lb_{2,2}}{\left(R_{A_{\kappa+r}}\right)^2 + (\omega_n)^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{1,\kappa+r}\right)^2\right]$$

$$L_{22C_n} := Lb_{2,2} - \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot Lb_{2,2}}{\left(R_{A_{\kappa+r}}\right)^2 + (\omega_n)^2\cdot\left(Lb_{2,2}\right)^2}\cdot\left(M_{2,\kappa+r}\right)^2\right] - \sum_{\kappa=1}^{2}\left[\frac{(\omega_n)^2\cdot Lb_{1,1}}{\left(R_{A_\kappa}\right)^2 + (\omega_n)^2\cdot\left(Lb_{1,1}\right)^2}\cdot\left(M_{2,\kappa}\right)^2\right]$$

Compute the leakage inductances.

Compute an array of the system impedance matrices.

$$Zm_n := j \cdot \omega_n \cdot M \qquad Za_n := R_A + j \cdot \omega_n \cdot L_A$$

$$Zsys_n := \begin{bmatrix} (Zb_n)_{1,1} & (Zb_n)_{1,2} & (Zm_n)_{1,1} & (Zm_n)_{1,2} & (Zm_n)_{1,3} & (Zm_n)_{1,4} \\ (Zb_n)_{2,1} & (Zb_n)_{2,2} & (Zm_n)_{2,1} & (Zm_n)_{2,2} & (Zm_n)_{2,3} & (Zm_n)_{2,4} \\ (Zm_n)_{1,1} & (Zm_n)_{2,1} & (Za_n)_1 & 0 & 0 & 0 \\ (Zm_n)_{1,2} & (Zm_n)_{2,2} & 0 & (Za_n)_2 & 0 & 0 \\ (Zm_n)_{1,3} & (Zm_n)_{2,3} & 0 & 0 & (Za_n)_3 & 0 \\ (Zm_n)_{1,4} & (Zm_n)_{2,4} & 0 & 0 & 0 & (Za_n)_4 \end{bmatrix}$$

Compute the winding currents at each frequency when 1 volt is applied to winding 1, and winding 2 is shorted.

$$Ileak12_n := (Zsys_n)^{-1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Compute the leakage impedances of winding 1 at each frequency.

$$Zleak12C_n := \frac{1}{(Ileak12_n)_1}$$

Compute the leakage resistances and inductances of winding 1 at each frequency.

$$Rleak12C_n := Re\left(Zleak12C_n\right) \qquad Lleak12C_n := \frac{Im\left(Zleak12C_n\right)}{\omega_n}$$

Compute the winding currents at each frequency when 1 volt is applied to winding 2, and winding 1 is shorted.

$$Ileak21_n := \left(Zsys_n\right)^{-1} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Compute the leakage impedances of winding 2 at each frequency.

$$Zleak21C_n := \frac{1}{\left(Ileak21_n\right)_2}$$

Compute the leakage resistances and inductances of winding 2 at each frequency.

$$Rleak21C_n := Re\left(Zleak21C_n\right) \qquad Lleak21C_n := \frac{Im\left(Zleak21C_n\right)}{\omega_n}$$

Compute the modeled self impedances

$$Z_{11C_n} := R_{11C_n} + j \cdot \omega_n \cdot L_{11C_n} \qquad Z_{22C_n} := R_{22C_n} + j \cdot \omega_n \cdot L_{22C_n}$$

Compute the modeled mutual impedances

$$Z_{12C_n} := \sqrt{\left(Z_{11C_n} - \mathrm{Zleak12C}_n\right) \cdot Z_{22C_n}}$$

Compute the modeled mutual resistances and inductances.

$$R_{12C_n} := \mathrm{Re}\left(Z_{12C_n}\right) \qquad L_{12C_n} := \frac{\mathrm{Im}\left(Z_{12C_n}\right)}{\omega_n}$$

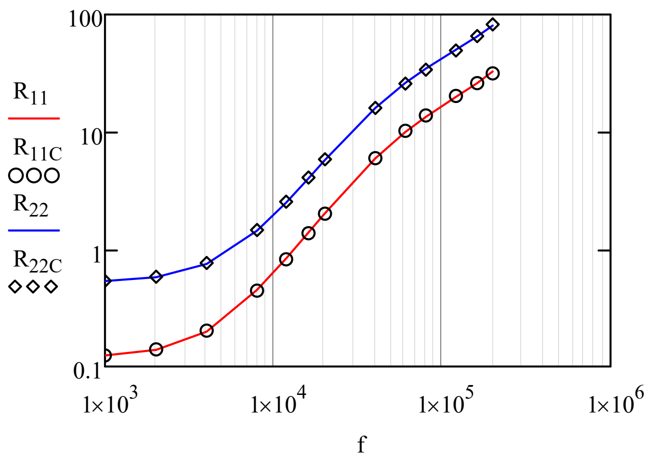

Fig. 13. Measured and calculated winding Resistances.

$R_{11C} =$

	1
1	0.1242
2	0.1400
3	0.2019
4	0.4432
5	0.8209
6	1.3685
7	2.0056
8	5.9673
9	10.1879
10	13.8044
11	20.2041
12	25.9928
13	31.5182

$R_{22C} =$

	1
1	0.5360
2	0.5820
3	0.7616
4	1.4579
5	2.5365
6	4.0755
7	5.8303
8	15.9671
9	25.7430
10	33.8825
11	49.3632
12	65.5023
13	82.4172

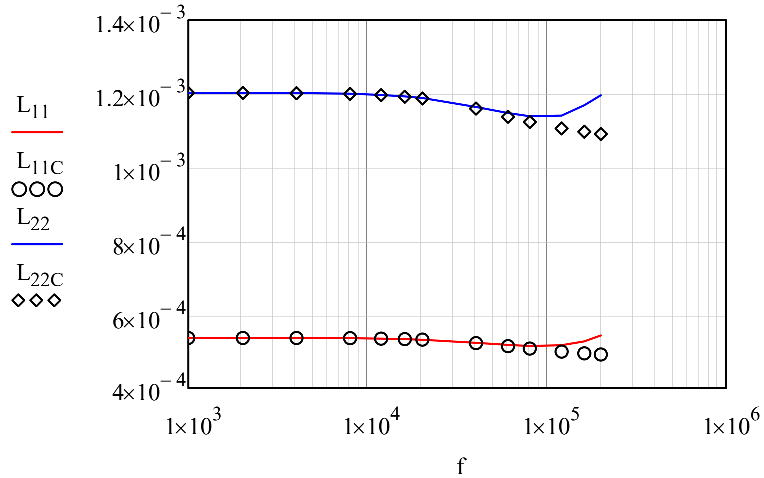

Fig. 14. Measured and calculated winding Inductances.

L_{11C} =

	1
1	$536.6967\cdot10^{-6}$
2	$536.6558\cdot10^{-6}$
3	$536.4961\cdot10^{-6}$
4	$535.8765\cdot10^{-6}$
5	$534.9143\cdot10^{-6}$
6	$533.5365\cdot10^{-6}$
7	$531.9594\cdot10^{-6}$
8	$522.7865\cdot10^{-6}$
9	$514.2286\cdot10^{-6}$
10	$507.9455\cdot10^{-6}$
11	$499.5174\cdot10^{-6}$
12	$494.7682\cdot10^{-6}$
13	$491.9365\cdot10^{-6}$

L_{22C} =

	1
1	$1.2026\cdot10^{-3}$
2	$1.2025\cdot10^{-3}$
3	$1.2020\cdot10^{-3}$
4	$1.1999\cdot10^{-3}$
5	$1.1967\cdot10^{-3}$
6	$1.1922\cdot10^{-3}$
7	$1.1872\cdot10^{-3}$
8	$1.1600\cdot10^{-3}$
9	$1.1380\cdot10^{-3}$
10	$1.1237\cdot10^{-3}$
11	$1.1063\cdot10^{-3}$
12	$1.0968\cdot10^{-3}$
13	$1.0907\cdot10^{-3}$

Note that the measured winding inductances do not fall monotonically as they do in the model because of resonance with the winding capacitances.

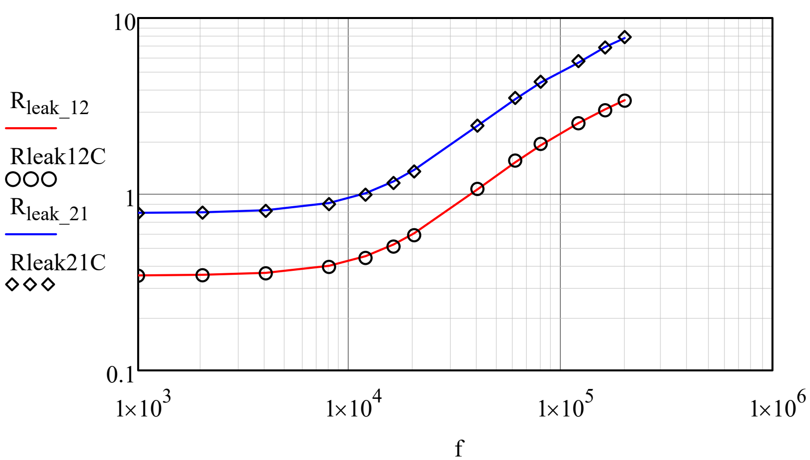

Fig. 15 Measured and computed leakage resistances.

Rleak12C =

	1
1	0.3442
2	0.3471
3	0.3552
4	0.3863
5	0.4348
6	0.5048
7	0.5856
8	1.0718
9	1.5520
10	1.9307
11	2.5264
12	2.9922
13	3.3942

Rleak21C =

	1
1	0.7811
2	0.7862
3	0.8052
4	0.8788
5	0.9933
6	1.1575
7	1.3459
8	2.4515
9	3.5152
10	4.3512
11	5.6981
12	6.8014
13	7.7872

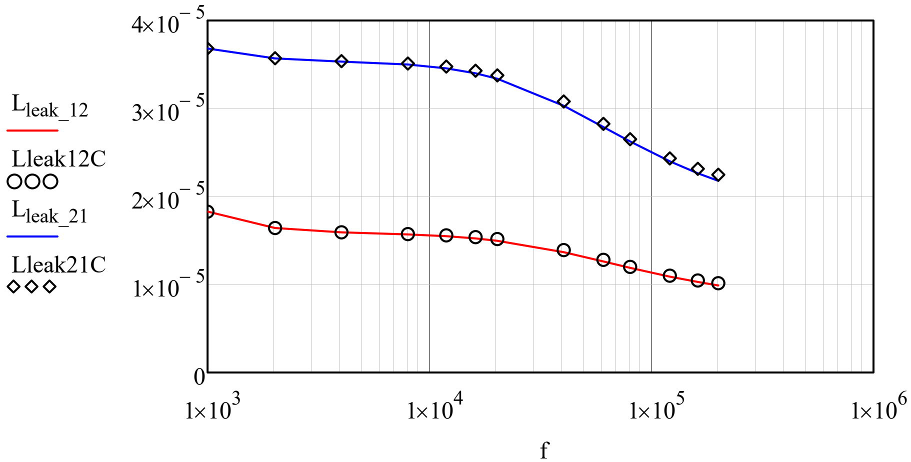

Fig. 16. Measured and computed leakage inductances.

Lleak12C =

	1
1	$18.2720 \cdot 10^{-6}$
2	$16.4099 \cdot 10^{-6}$
3	$15.9340 \cdot 10^{-6}$
4	$15.7344 \cdot 10^{-6}$
5	$15.5811 \cdot 10^{-6}$
6	$15.3839 \cdot 10^{-6}$
7	$15.1640 \cdot 10^{-6}$
8	$13.9131 \cdot 10^{-6}$
9	$12.7912 \cdot 10^{-6}$
10	$12.0032 \cdot 10^{-6}$
11	$10.9964 \cdot 10^{-6}$
12	$10.4582 \cdot 10^{-6}$
13	$10.1530 \cdot 10^{-6}$

Lleak21C =

	1
1	$36.8128 \cdot 10^{-6}$
2	$35.7040 \cdot 10^{-6}$
3	$35.3824 \cdot 10^{-6}$
4	$35.1027 \cdot 10^{-6}$
5	$34.7619 \cdot 10^{-6}$
6	$34.2902 \cdot 10^{-6}$
7	$33.7585 \cdot 10^{-6}$
8	$30.7973 \cdot 10^{-6}$
9	$28.2514 \cdot 10^{-6}$
10	$26.5121 \cdot 10^{-6}$
11	$24.3207 \cdot 10^{-6}$
12	$23.1454 \cdot 10^{-6}$
13	$22.4675 \cdot 10^{-6}$

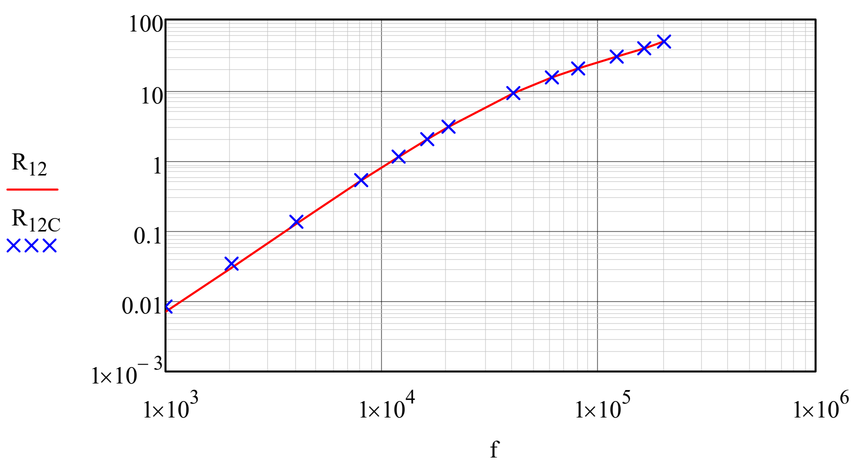

Fig. 17. Measured and modeled mutual resistances.

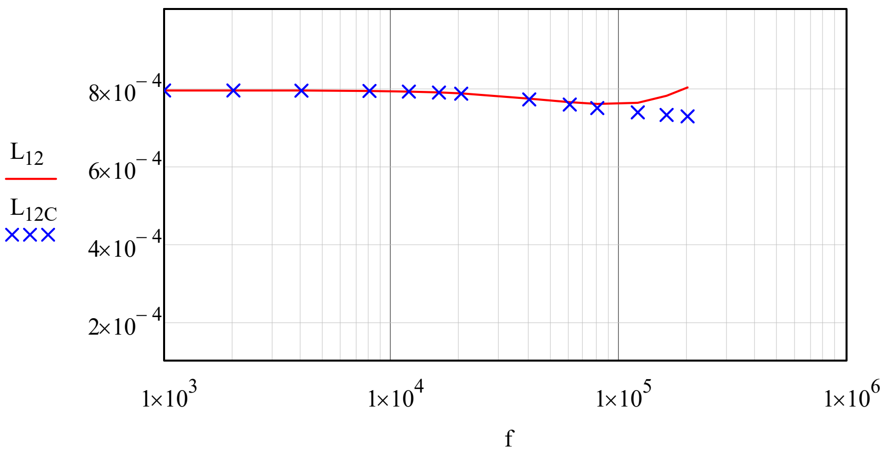

Fig. 18. Measured and modeled mutual inductances.

Check eigenvalues of the system inductance matrix to ensure that the system is stable. All of the eigenvalues should be positive. See [2].

$$L_{sys} := \frac{\mathrm{Im}\left(Zsys_1\right)}{\omega_1} \qquad \mathrm{eigenvals}\left(L_{sys}\right) = \begin{pmatrix} 3.4960 \times 10^{-6} \\ 3.8801 \times 10^{-4} \\ 5.3959 \times 10^{-4} \\ 9.8572 \times 10^{-4} \\ 1.2027 \times 10^{-3} \\ 2.0986 \times 10^{-3} \end{pmatrix}$$

Compute coupling coefficient matrix to get an idea of how close the couplings are.

$$K_{sys} := \begin{pmatrix} 1 & \frac{L_{sys_{1,2}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{2,2}}}} & \frac{L_{sys_{1,3}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{3,3}}}} & \frac{L_{sys_{1,4}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{4,4}}}} & \frac{L_{sys_{1,5}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{5,5}}}} & \frac{L_{sys_{1,6}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{6,6}}}} \\ \frac{L_{sys_{1,2}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{2,2}}}} & 1 & \frac{L_{sys_{2,3}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{3,3}}}} & \frac{L_{sys_{2,4}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{4,4}}}} & \frac{L_{sys_{2,5}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{5,5}}}} & \frac{L_{sys_{2,6}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{6,6}}}} \\ \frac{L_{sys_{1,3}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{3,3}}}} & \frac{L_{sys_{2,3}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{3,3}}}} & 1 & 0 & 0 & 0 \\ \frac{L_{sys_{1,4}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{4,4}}}} & \frac{L_{sys_{2,4}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{4,4}}}} & 0 & 1 & 0 & 0 \\ \frac{L_{sys_{1,5}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{5,5}}}} & \frac{L_{sys_{2,5}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{5,5}}}} & 0 & 0 & 1 & 0 \\ \frac{L_{sys_{1,6}}}{\sqrt{L_{sys_{1,1}} \cdot L_{sys_{6,6}}}} & \frac{L_{sys_{2,6}}}{\sqrt{L_{sys_{2,2}} \cdot L_{sys_{6,6}}}} & 0 & 0 & 0 & 1 \end{pmatrix}$$

Check the eigenvalues of coupling coefficient matrix. They should all be positive.

$$K_{sys} = \begin{pmatrix} 1.0000 & 0.9852 & 0.2358 & 0.3325 & 0.1854 & 0.1936 \\ 0.9852 & 1.0000 & 0.1611 & 0.3796 & 0.2627 & 0.2629 \\ 0.2358 & 0.1611 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.3325 & 0.3796 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.1854 & 0.2627 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.1936 & 0.2629 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \qquad \mathrm{eigenvals}(K_{sys}) = \begin{pmatrix} 4.6202 \times 10^{-3} \\ 0.6125 \\ 1.0000 \\ 1.0000 \\ 1.0071 \\ 2.3758 \end{pmatrix}$$

Mutual resistance coupling. These results are not used in the calculations, but are included to show the relative levels of loss coupling among the windings.

$$kr_{12} := \overrightarrow{\frac{R_{12}}{\sqrt{R_{11} \cdot R_{22}}}} \qquad kr_{12C} := \overrightarrow{\frac{R_{12C}}{\sqrt{R_{11C} \cdot R_{22C}}}}$$

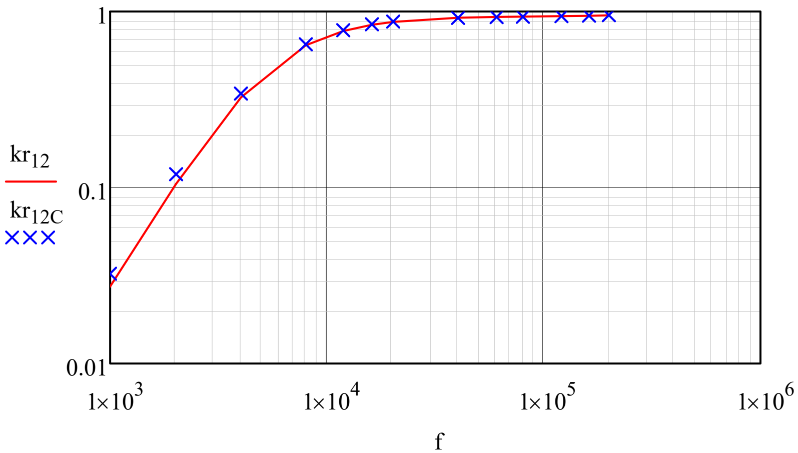

Fig. 19. Measured and modeled mutual resistance couplings.

$kr_{12} =$

	1
1	0.0273
2	0.1049
3	0.3247
4	0.6431
5	0.7742
6	0.8401
7	0.8723
8	0.9209
9	0.9317
10	0.9360
11	0.9414
12	0.9457
13	0.9513

$kr_{12C} =$

	1
1	0.0324
2	0.1190
3	0.3420
4	0.6507
5	0.7820
6	0.8462
7	0.8771
8	0.9203
9	0.9292
10	0.9336
11	0.9405
12	0.9468
13	0.9525

SPICE model values

$L1 := Lb_{1,1} \cdot H = 536.71 \cdot \mu H$ $\quad LA11 := L1 \quad LA12 := L1$

$L2 := Lb_{2,2} \cdot H = 1.2027 \cdot mH$ $\quad LA21 := L2 \quad LA22 := L2$

$Kps := K_{sys_{2,1}} = 0.98518$

$R_{A11} := R_{A_1} = 277.5980$

$R_{A12} := R_{A_2} = 6.7350 \times 10^3$

$R_{A21} := R_{A_3} = 344.5523$

$R_{A22} := R_{A_4} = 6.2452 \times 10^3$

$K11 := K_{sys_{1,3}} = 0.23584 \quad L_1 \quad L_{A11}$

$K12 := K_{sys_{1,4}} = 0.33250 \quad L_1 \quad L_{A12}$

$K13 := K_{sys_{1,5}} = 0.185439 \quad L_1 \quad L_{A21}$

$K14 := K_{sys_{1,6}} = 0.19356 \quad L_1 \quad L_{A22}$

$K21 := K_{sys_{2,3}} = 0.16114 \quad L_2 \quad L_{A11}$

$K22 := K_{sys_{2,4}} = 0.37963 \quad L_2 \quad L_{A12}$

$K23 := K_{sys_{2,5}} = 0.26272 \quad L_2 \quad L_{A21}$

$K24 := K_{sys_{2,6}} = 0.26292 \quad L_2 \quad L_{A22}$

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[2] Yilmaz Tokad and Myril B. Reed, "Criteria and Tests for Realizability of the Inductance Matrix,"
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[3] J. H. Spreen, "Electrical terminal representation of conductor loss in transformers," in IEEE Transactions on Power Electronics, vol. 5, no. 4, pp. 424-429, Oct. 1990.
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