Four-Winding Transformer Model Coefficient Extraction from Measured Data

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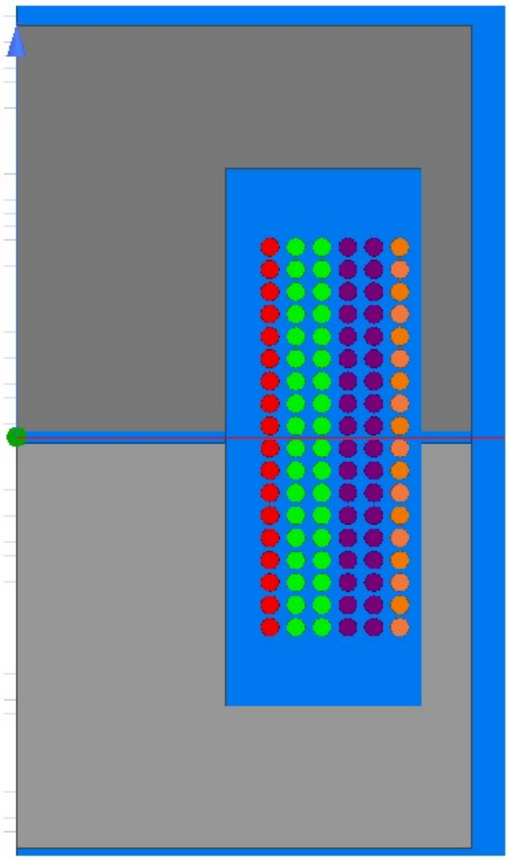


All wires 0.28mm single insulation 155C.
(A little smaller than 30 AWG)

Primary: 18T with 1st and last layers in parallel.

Secondaries: 36 turns each.

3.5 mil tape between each layer.

Core: B65807J0000R049 RM6 N49

Gap: 8 mil

Bobbin: B65808E1508T001

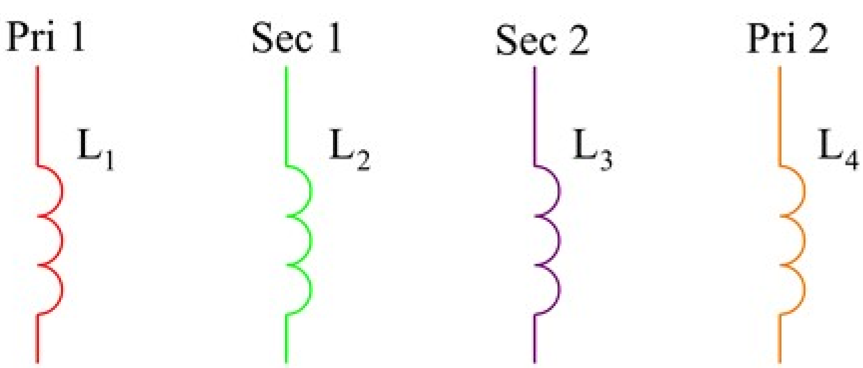


Fig. 1. Transformer construction details.

Unit definitions. $m\Omega := \frac{\Omega}{1000}$ $nH := H \cdot 10^{-9}$

Measured dc resistances. $Rdc := \begin{bmatrix} 0.1716 \\ 0.3758 \\ 0.4250 \\ .2406 \end{bmatrix} \cdot \Omega$

Low-frequency resistances and inductances.

$R0 := \begin{bmatrix} 0.172600654 \\ 0.382819163 \\ 0.435272334 \\ 0.239045168 \end{bmatrix} \cdot \Omega$ $L0 := \begin{bmatrix} 4.55376 \cdot 10^{-5} \\ 1.82487 \cdot 10^{-4} \\ 1.84187 \cdot 10^{-4} \\ 4.65797 \cdot 10^{-5} \end{bmatrix} \cdot H$ $f0 := 10 \cdot kHz$ $\omega 0 := 2 \cdot \pi \cdot f0$

$$Rb := \begin{bmatrix} Rdc_1 & 0 & 0 & 0 \\ 0 & Rdc_2 & 0 & 0 \\ 0 & 0 & Rdc_3 & 0 \\ 0 & 0 & 0 & Rdc_4 \end{bmatrix}$$

Low-frequency leakage inductances.

$Lleak12\_0 := 0.000000759198 \cdot H$ $Lleak13\_0 := 0.00000192703 \cdot H$ $Lleak14\_0 := 0.00000326471 \cdot H$

$Lleak23\_0 := 0.00000386677 \cdot H$ $Lleak24\_0 := 0.00000935959 \cdot H$ $Lleak34\_0 := 0.00000470411 \cdot H$

$$Kb := \begin{bmatrix} 1 & \sqrt{1 - \frac{Lleak12\_0}{L0_1}} & \sqrt{1 - \frac{Lleak13\_0}{L0_1}} & \sqrt{1 - \frac{Lleak14\_0}{L0_1}} \\ \sqrt{1 - \frac{Lleak12\_0}{L0_1}} & 1 & \sqrt{1 - \frac{Lleak23\_0}{L0_2}} & \sqrt{1 - \frac{Lleak24\_0}{L0_2}} \\ \sqrt{1 - \frac{Lleak13\_0}{L0_1}} & \sqrt{1 - \frac{Lleak23\_0}{L0_2}} & 1 & \sqrt{1 - \frac{Lleak34\_0}{L0_3}} \\ \sqrt{1 - \frac{Lleak14\_0}{L0_1}} & \sqrt{1 - \frac{Lleak24\_0}{L0_2}} & \sqrt{1 - \frac{Lleak34\_0}{L0_3}} & 1 \end{bmatrix}$$

$$Kb = \begin{bmatrix} 1.0000 & 0.9916 & 0.9786 & 0.9635 \\ 0.9916 & 1.0000 & 0.9893 & 0.9740 \\ 0.9786 & 0.9893 & 1.0000 & 0.9871 \\ 0.9635 & 0.9740 & 0.9871 & 1.0000 \end{bmatrix}$$

All of the eigenvalues must be positive to have a stable model [2]. $\mathrm{eigenvals}(Kb) = \begin{bmatrix} 3.942 \\ 0.042 \\ 0.011 \\ 0.005 \end{bmatrix}$

$$Lb := \begin{bmatrix} L0_1 & Kb_{1,2} \cdot \sqrt{L0_1 \cdot L0_2} & Kb_{1,3} \cdot \sqrt{L0_1 \cdot L0_3} & Kb_{1,4} \cdot \sqrt{L0_1 \cdot L0_4} \\ Kb_{1,2} \cdot \sqrt{L0_1 \cdot L0_2} & L0_2 & Kb_{2,3} \cdot \sqrt{L0_2 \cdot L0_3} & Kb_{2,4} \cdot \sqrt{L0_2 \cdot L0_4} \\ Kb_{1,3} \cdot \sqrt{L0_1 \cdot L0_3} & Kb_{2,3} \cdot \sqrt{L0_2 \cdot L0_3} & L0_3 & Kb_{3,4} \cdot \sqrt{L0_3 \cdot L0_4} \\ Kb_{1,4} \cdot \sqrt{L0_1 \cdot L0_4} & Kb_{2,4} \cdot \sqrt{L0_2 \cdot L0_4} & Kb_{3,4} \cdot \sqrt{L0_3 \cdot L0_4} & L0_4 \end{bmatrix}$$

$$Lb = \begin{bmatrix} 4.554 \cdot 10^{-5} & 9.04 \cdot 10^{-5} & 8.962 \cdot 10^{-5} & 4.437 \cdot 10^{-5} \\ 9.04 \cdot 10^{-5} & 1.825 \cdot 10^{-4} & 1.814 \cdot 10^{-4} & 8.98 \cdot 10^{-5} \\ 8.962 \cdot 10^{-5} & 1.814 \cdot 10^{-4} & 1.842 \cdot 10^{-4} & 9.143 \cdot 10^{-5} \\ 4.437 \cdot 10^{-5} & 8.98 \cdot 10^{-5} & 9.143 \cdot 10^{-5} & 4.658 \cdot 10^{-5} \end{bmatrix} \boldsymbol{H}$$

All of the eigenvalues must be positive to have a stable model [2]. $\mathrm{eigenvals}(Lb) = \begin{bmatrix} 4.544 \cdot 10^{-4} \\ 3.035 \cdot 10^{-6} \\ 8.56 \cdot 10^{-7} \\ 5.287 \cdot 10^{-7} \end{bmatrix} \boldsymbol{H}$

Measured self resistances and inductances.

freq (*Hz*)	*R11* (*ohm*)	*L11* (*H*)	*R44* (*ohm*)	*L44* (*H*)
20417.379	0.176714	0.0000454685	0.244835	0.0000465008
69183.097	0.204333	0.0000454741	0.294545	0.0000464998
123026.877	0.270488	0.0000454485	0.410325	0.0000464591
173780.083	0.354768	0.0000454256	0.564212	0.0000464175
234422.882	0.474137	0.0000453944	0.781677	0.0000463579
275422.87	0.558997	0.0000453741	0.941292	0.0000463162
363078.055	0.765276	0.0000453339	1.315280	0.0000462278
416869.383	0.903397	0.0000453100	1.557840	0.0000461734
512861.384	1.156536	0.0000452664	1.973551	0.0000460801
562341.325	1.288912	0.0000452459	2.188143	0.0000460344
741310.241	1.773047	0.0000451798	2.877101	0.0000458940
1230268.771	3.533518	0.0000451024	4.804675	0.0000456974
1995262.315	12.171316	0.0000450339	13.050004	0.0000455485

R22 (*ohm*)	*L22* (*H*)	*R33* (*ohm*)	*L33* (*H*)
0.464219	0.0001838580	0.408705	0.0001821664
0.648541	0.0001837672	0.553922	0.0001821136
1.034821	0.0001836550	0.856800	0.0001820252
1.545904	0.0001835411	1.255194	0.0001819374
2.269451	0.0001833607	1.819483	0.0001818064
2.784273	0.0001832497	2.233939	0.0001817188
4.088262	0.0001830052	3.234132	0.0001815572
4.857854	0.0001830560	3.836210	0.0001816473
6.323104	0.0001828094	5.036927	0.0001814737
7.072642	0.0001826978	5.654723	0.0001813946
9.683816	0.0001824044	7.912521	0.0001811981
18.124881	0.0001824010	15.934445	0.0001813085
53.733561	0.0001828725	51.988373	0.0001817767

$f := 1 .. \mathrm{rows}(freq)$ $\qquad$ $\omega_f := 2 \cdot \pi \cdot freq_f$

Compute the self impedances from the measured resistance and inductance values.

$Z11_f := R11_f + 1\mathrm{j} \cdot \omega_f \cdot L11_f$ $\qquad$ $Z22_f := R22_f + 1\mathrm{j} \cdot \omega_f \cdot L22_f$

$Z33_f := R33_f + 1\mathrm{j} \cdot \omega_f \cdot L33_f$ $\qquad$ $Z44_f := R44_f + 1\mathrm{j} \cdot \omega_f \cdot L44_f$

Measured Leakage Impedance Data

Rleak12 (ohm)	Lleak12 (H)
0.264212	$7.204710 \cdot 10^{-7}$
0.276394	$7.095860 \cdot 10^{-7}$
0.304332	$7.011700 \cdot 10^{-7}$
0.342738	$6.912680 \cdot 10^{-7}$
0.399902	$6.765250 \cdot 10^{-7}$
0.443340	$6.655380 \cdot 10^{-7}$
0.542981	$6.407060 \cdot 10^{-7}$
0.605153	$6.255100 \cdot 10^{-7}$
0.711963	$5.999310 \cdot 10^{-7}$
0.763490	$5.878880 \cdot 10^{-7}$
0.928093	$5.512740 \cdot 10^{-7}$
1.246240	$4.930080 \cdot 10^{-7}$
1.583520	$4.539470 \cdot 10^{-7}$

Rleak13 (ohm)	Lleak13 (H)
0.273733	$1.878550 \cdot 10^{-6}$
0.310390	$1.859750 \cdot 10^{-6}$
0.394293	$1.836980 \cdot 10^{-6}$
0.509181	$1.807410 \cdot 10^{-6}$
0.680391	$1.763700 \cdot 10^{-6}$
0.810365	$1.730750 \cdot 10^{-6}$
1.108310	$1.656240 \cdot 10^{-6}$
1.293740	$1.610430 \cdot 10^{-6}$
1.611570	$1.533150 \cdot 10^{-6}$
1.764610	$1.496570 \cdot 10^{-6}$
2.246890	$1.384960 \cdot 10^{-6}$
3.126450	$1.211190 \cdot 10^{-6}$
3.982920	$1.102930 \cdot 10^{-6}$

Rleak14 (ohm)	Lleak14 (H)
0.395800	$3.044960 \cdot 10^{-6}$
0.452577	$2.970660 \cdot 10^{-6}$
0.581019	$2.932590 \cdot 10^{-6}$
0.756703	$2.887400 \cdot 10^{-6}$
1.019400	$2.821410 \cdot 10^{-6}$
1.219700	$2.771760 \cdot 10^{-6}$
1.681240	$2.659070 \cdot 10^{-6}$
1.970640	$2.589080 \cdot 10^{-6}$
2.469180	$2.470870 \cdot 10^{-6}$
2.711730	$2.414470 \cdot 10^{-6}$
3.479890	$2.242010 \cdot 10^{-6}$
4.901140	$1.970070 \cdot 10^{-6}$
6.316170	$1.803390 \cdot 10^{-6}$

Rleak23 (ohm)	Lleak23 (H)
0.790386	$3.672360 \cdot 10^{-6}$
0.862053	$3.605510 \cdot 10^{-6}$
1.025280	$3.558750 \cdot 10^{-6}$
1.248250	$3.500270 \cdot 10^{-6}$
1.579770	$3.414520 \cdot 10^{-6}$
1.831180	$3.350180 \cdot 10^{-6}$
2.405310	$3.205110 \cdot 10^{-6}$
2.762360	$3.116500 \cdot 10^{-6}$
3.372790	$2.967710 \cdot 10^{-6}$
3.666640	$2.897510 \cdot 10^{-6}$
4.595220	$2.684400 \cdot 10^{-6}$
6.314380	$2.351830 \cdot 10^{-6}$
8.038020	$2.142680 \cdot 10^{-6}$

Rleak24 (ohm)	Lleak24 (H)
1.271310	$8.482470 \cdot 10^{-6}$
1.432770	$8.198840 \cdot 10^{-6}$
1.799240	$8.086560 \cdot 10^{-6}$
2.299350	$7.954210 \cdot 10^{-6}$
3.045390	$7.764190 \cdot 10^{-6}$
3.612440	$7.621520 \cdot 10^{-6}$
4.913050	$7.299060 \cdot 10^{-6}$
5.724080	$7.101200 \cdot 10^{-6}$
7.118420	$6.766650 \cdot 10^{-6}$
7.792920	$6.607940 \cdot 10^{-6}$
9.927260	$6.124990 \cdot 10^{-6}$
13.877300	$5.372270 \cdot 10^{-6}$
17.870600	$4.918530 \cdot 10^{-6}$

Rleak34 (ohm)	Lleak34 (H)
1.344110	$3.799260 \cdot 10^{-6}$
1.405750	$3.528830 \cdot 10^{-6}$
1.542570	$3.474950 \cdot 10^{-6}$
1.729030	$3.423110 \cdot 10^{-6}$
2.006730	$3.351100 \cdot 10^{-6}$
2.217880	$3.297880 \cdot 10^{-6}$
2.703150	$3.178250 \cdot 10^{-6}$
3.006660	$3.105190 \cdot 10^{-6}$
3.531380	$2.982530 \cdot 10^{-6}$
3.786710	$2.924470 \cdot 10^{-6}$
4.609630	$2.747150 \cdot 10^{-6}$
6.234380	$2.465260 \cdot 10^{-6}$
8.030120	$2.282450 \cdot 10^{-6}$

Compute the leakage impedances from the measured leakage resistance and inductance values.

$$Zleak12_f := Rleak12_f + 1j \cdot \omega_f \cdot Lleak12_f \qquad Zleak13_f := Rleak13_f + 1j \cdot \omega_f \cdot Lleak13_f$$

$$Zleak14_f := Rleak14_f + 1j \cdot \omega_f \cdot Lleak14_f \qquad Zleak23_f := Rleak23_f + 1j \cdot \omega_f \cdot Lleak23_f$$

$$Zleak24_f := Rleak24_f + 1j \cdot \omega_f \cdot Lleak24_f \qquad Zleak34_f := Rleak34_f + 1j \cdot \omega_f \cdot Lleak34_f$$

Compute the mutual impedances.

$$Z12_f := \sqrt{\left(Z11_f - Zleak12_f\right) \cdot Z22_f} \qquad Z13_f := \sqrt{\left(Z11_f - Zleak13_f\right) \cdot Z33_f}$$

$$Z14_f := \sqrt{\left(Z11_f - Zleak14_f\right) \cdot Z44_f} \qquad Z23_f := \sqrt{\left(Z22_f - Zleak23_f\right) \cdot Z33_f}$$

$$Z24_f := \sqrt{\left(Z22_f - Zleak24_f\right) \cdot Z44_f} \qquad Z34_f := \sqrt{\left(Z33_f - Zleak34_f\right) \cdot Z44_f}$$

Compute the mutual resistances.

$$R12_f := \mathrm{Re}\left(Z12_f\right) \qquad R13_f := \mathrm{Re}\left(Z13_f\right) \qquad R14_f := \mathrm{Re}\left(Z14_f\right)$$

$$R23_f := \mathrm{Re}\left(Z23_f\right) \qquad R24_f := \mathrm{Re}\left(Z24_f\right) \qquad R34_f := \mathrm{Re}\left(Z34_f\right)$$

Compute the mutual inductances.

$$L12_f := \frac{\mathrm{Im}\left(Z12_f\right)}{\omega_f} \qquad L13_f := \frac{\mathrm{Im}\left(Z13_f\right)}{\omega_f} \qquad L14_f := \frac{\mathrm{Im}\left(Z14_f\right)}{\omega_f}$$

$$L23_f := \frac{\mathrm{Im}\left(Z23_f\right)}{\omega_f} \qquad L24_f := \frac{\mathrm{Im}\left(Z24_f\right)}{\omega_f} \qquad L34_f := \frac{\mathrm{Im}\left(Z34_f\right)}{\omega_f}$$

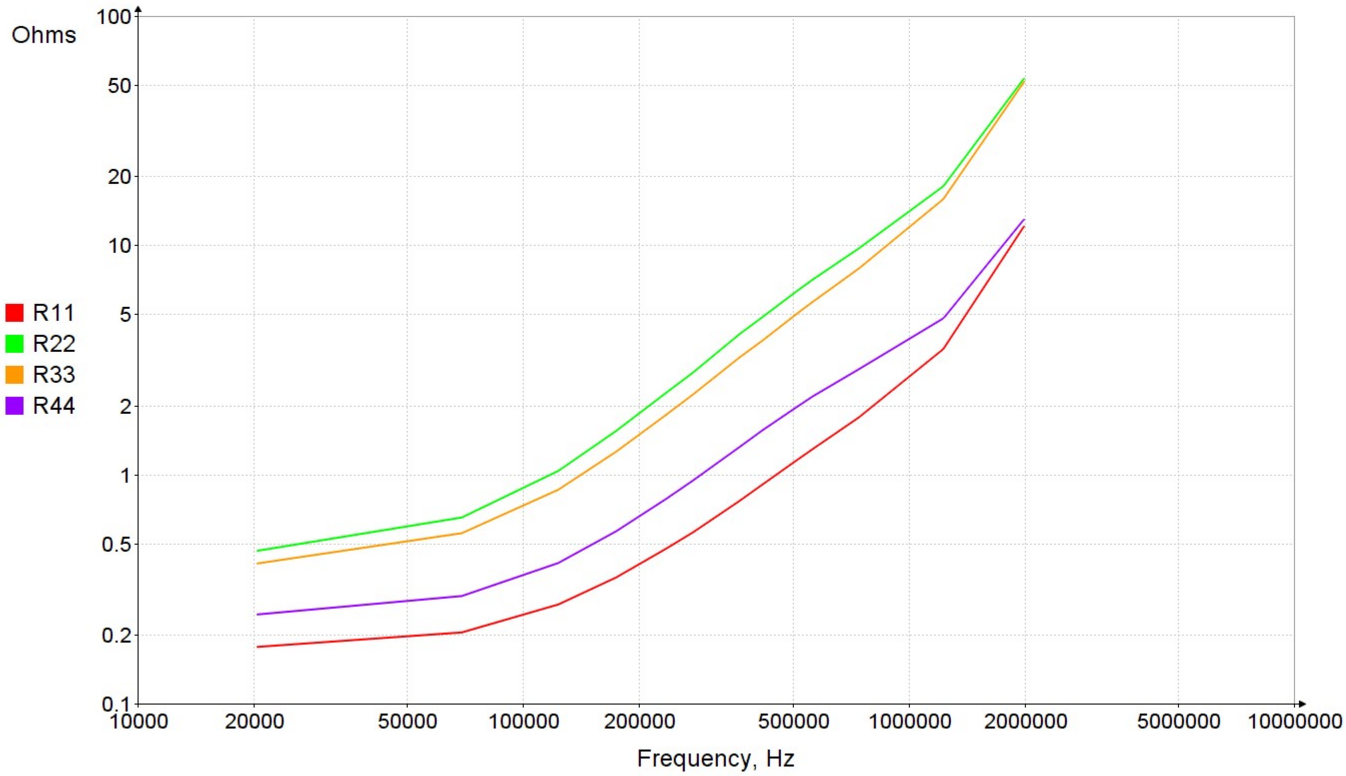


Fig. 2. Simulated self resistances.

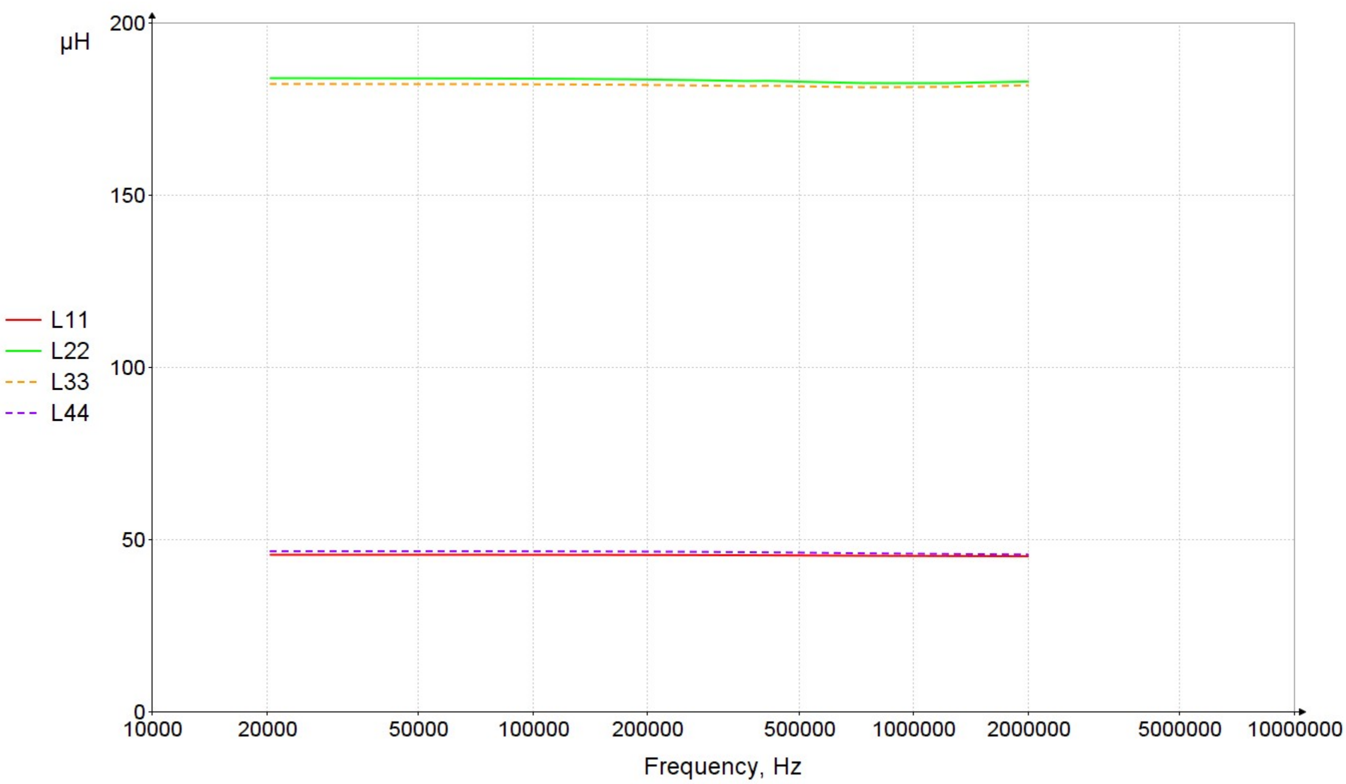


Fig. 3. Measured self inductances.

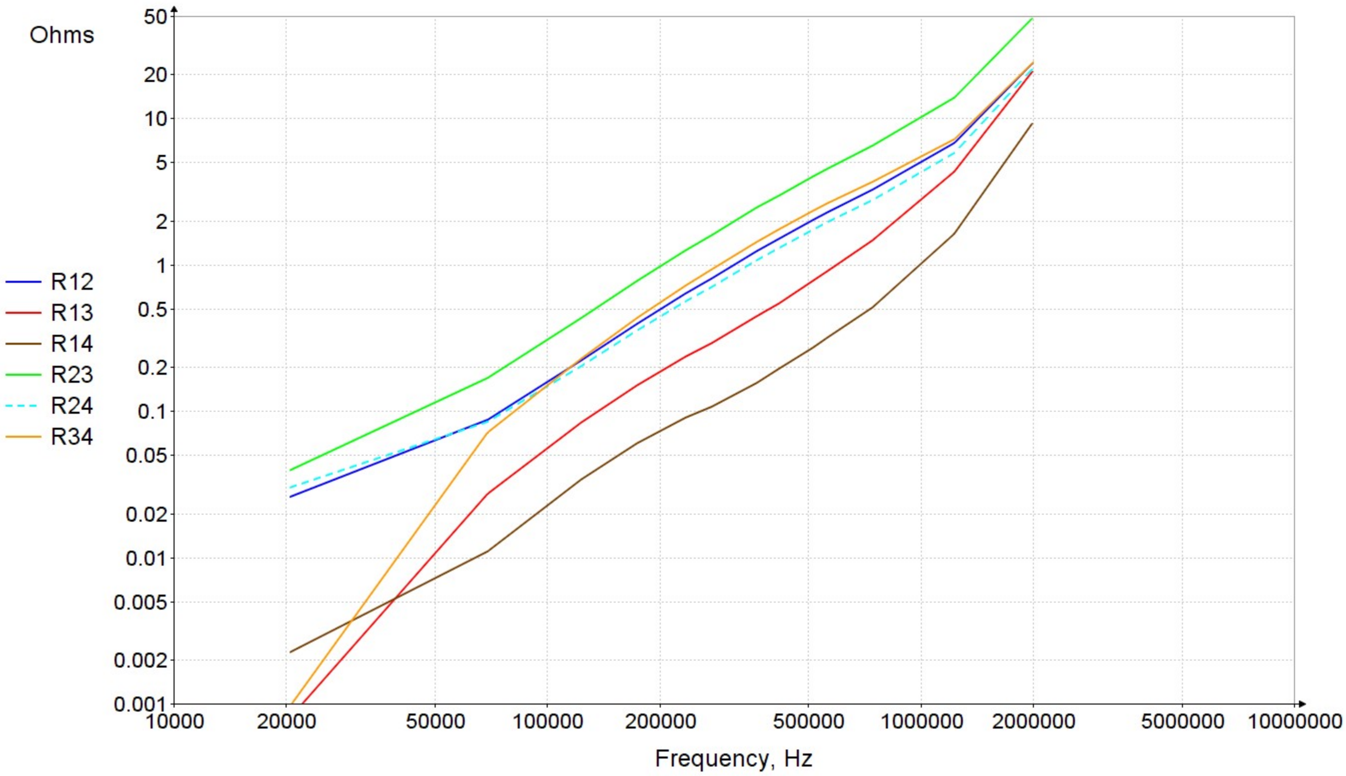


Fig. 4. Measured mutual resistances.

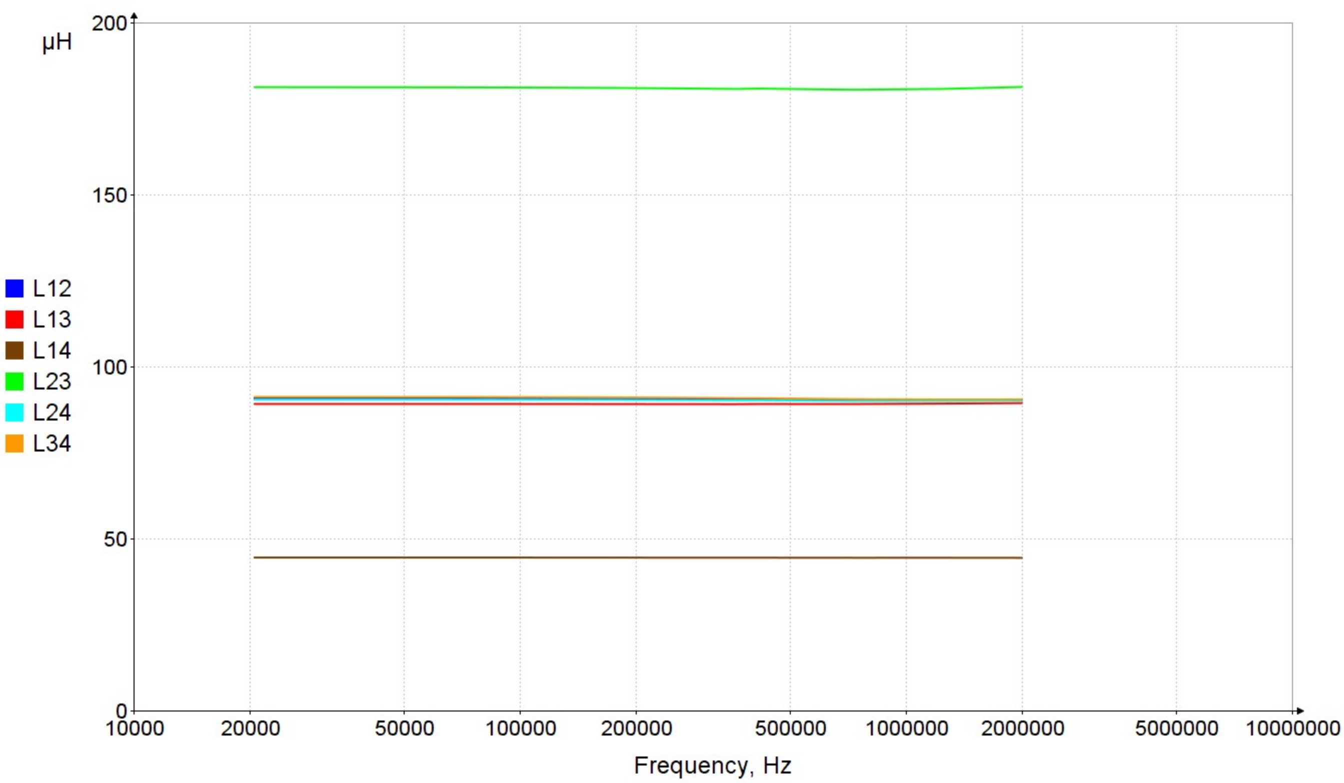


Fig. 5. Measured mutual inductances.

The imported simulation data is made available for subsequent calculation through functions that are indexed by the frequency of the matrix data.

$$RF(f) := \begin{bmatrix} R11_f \\ R22_f \\ R33_f \\ R44_f \\ R12_f \\ R13_f \\ R14_f \\ R23_f \\ R24_f \\ R34_f \end{bmatrix} \qquad LF(f) := \begin{bmatrix} L11_f \\ L22_f \\ L33_f \\ L44_f \\ L12_f \\ L13_f \\ L14_f \\ L23_f \\ L24_f \\ L34_f \end{bmatrix}$$

We begin the process of modeling the transformer by defining the voltages and currents as shown in Fig. 6.

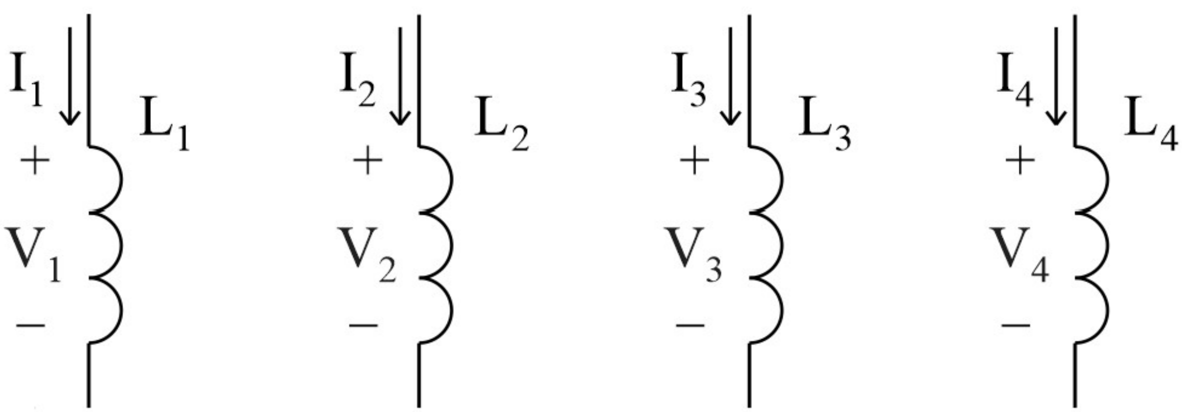


Fig. 6. Voltages and currents

As with any four-port network, the transformer voltages and currents can be described in the frequency domain in terms of self and mutual impedances.

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 + Z_{13} \cdot I_3 + Z_{14} \cdot I_3 \tag{1}$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2 + Z_{23} \cdot I_3 + Z_{24} \cdot I_3 \tag{2}$$

$$V_3 = Z_{31} \cdot I_1 + Z_{32} \cdot I_2 + Z_{33} \cdot I_3 + Z_{34} \cdot I_3 \tag{3}$$

$$V_4 = Z_{41} \cdot I_1 + Z_{42} \cdot I_2 + Z_{43} \cdot I_3 + Z_{44} \cdot I_4 \tag{4}$$

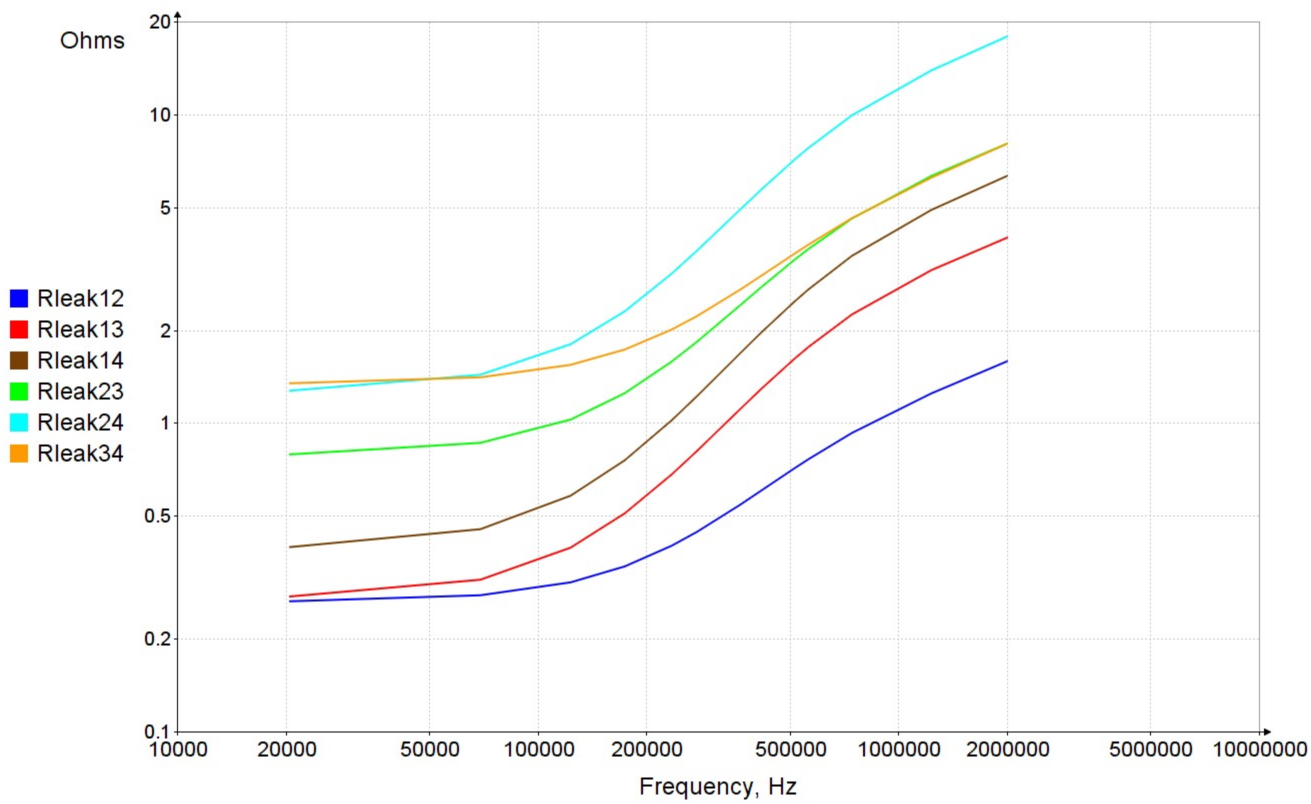


Fig. 7. Measured Leakage resistances.

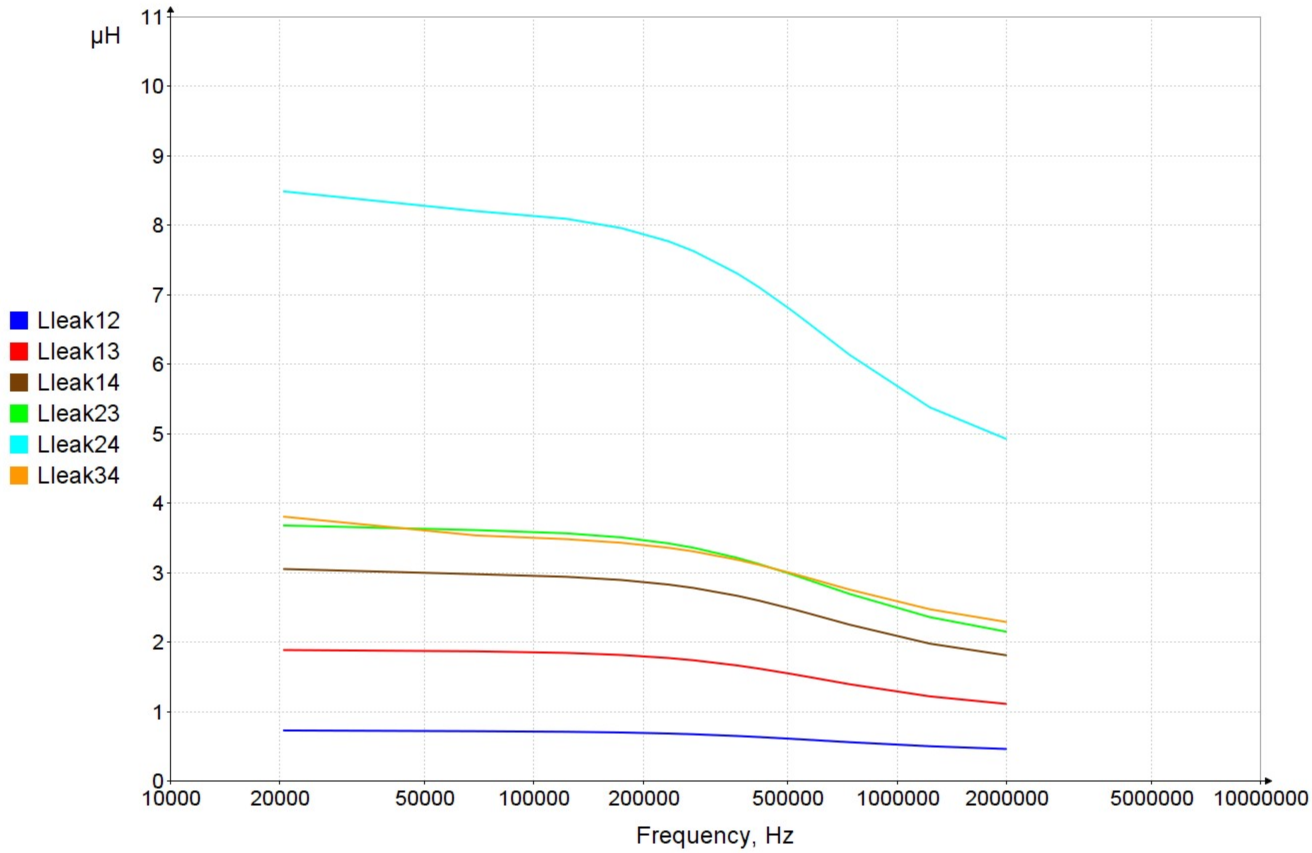
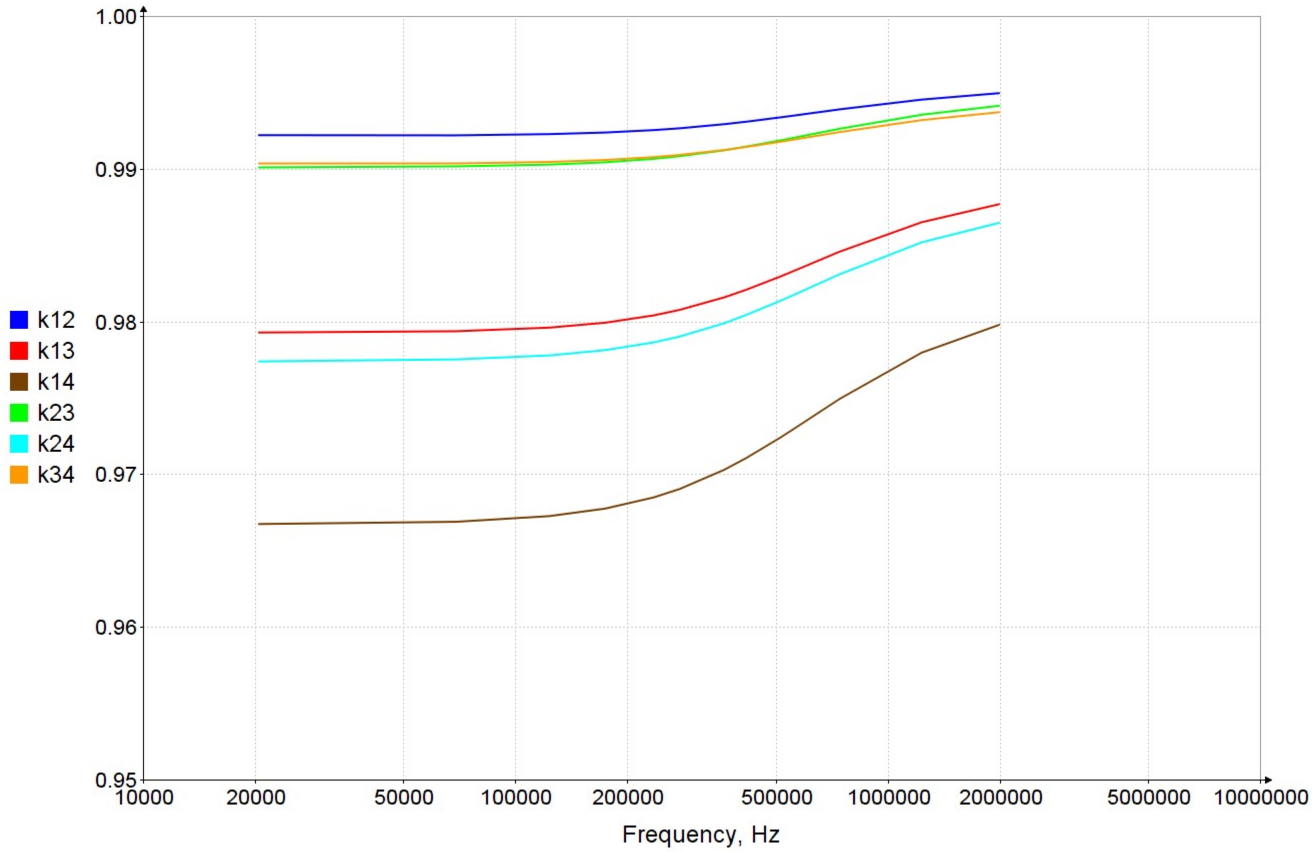


Fig. 8. Leakage inductances.

Mutual inductance couplings

$$k12 := \overrightarrow{\frac{L12}{\sqrt{L11 \cdot L22}}} \qquad k13 := \overrightarrow{\frac{L13}{\sqrt{L11 \cdot L33}}} \qquad k14 := \overrightarrow{\frac{L14}{\sqrt{L11 \cdot L44}}}$$

$$k23 := \overrightarrow{\frac{L23}{\sqrt{L22 \cdot L33}}} \qquad k24 := \overrightarrow{\frac{L24}{\sqrt{L22 \cdot L44}}} \qquad k34 := \overrightarrow{\frac{L34}{\sqrt{L33 \cdot L44}}}$$



Mutual Resistance couplings

$$kr12 := \overrightarrow{\frac{R12}{\sqrt{R11 \cdot R22}}} \qquad kr13 := \overrightarrow{\frac{R13}{\sqrt{R11 \cdot R33}}} \qquad kr14 := \overrightarrow{\frac{R14}{\sqrt{R11 \cdot R44}}}$$

$$kr23 := \overrightarrow{\frac{R23}{\sqrt{R22 \cdot R33}}} \qquad kr24 := \overrightarrow{\frac{R24}{\sqrt{R22 \cdot R44}}} \qquad kr34 := \overrightarrow{\frac{R34}{\sqrt{R33 \cdot R44}}}$$

Mutual Resistance Function

$$KR(f) := \begin{bmatrix} kr12_f \\ kr13_f \\ kr14_f \\ kr23_f \\ kr24_f \\ kr34_f \end{bmatrix}$$

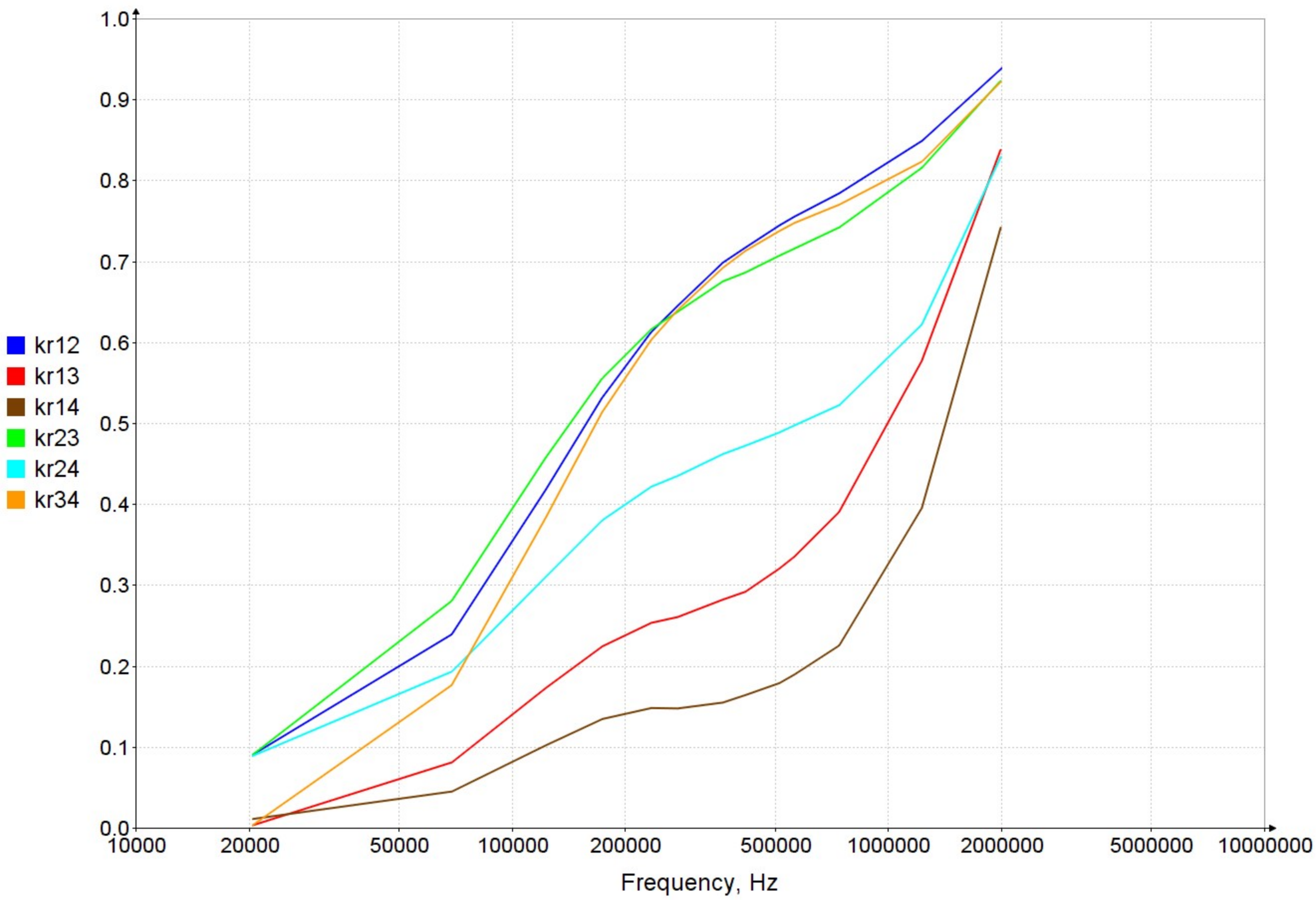
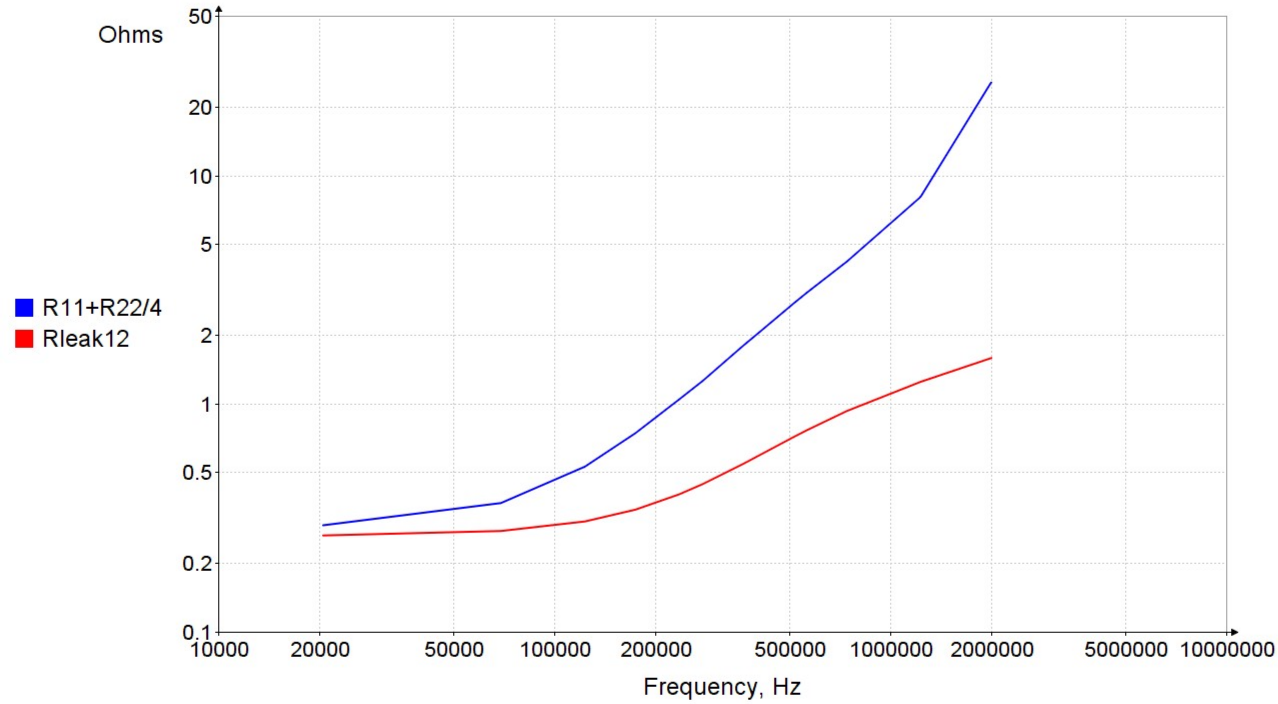


Fig. 9. Mutual Resistance Couplings.

N1 = 18 Turns, N2 = 36 Turns, N3 = 36 Turns, N4 = 18 Turns



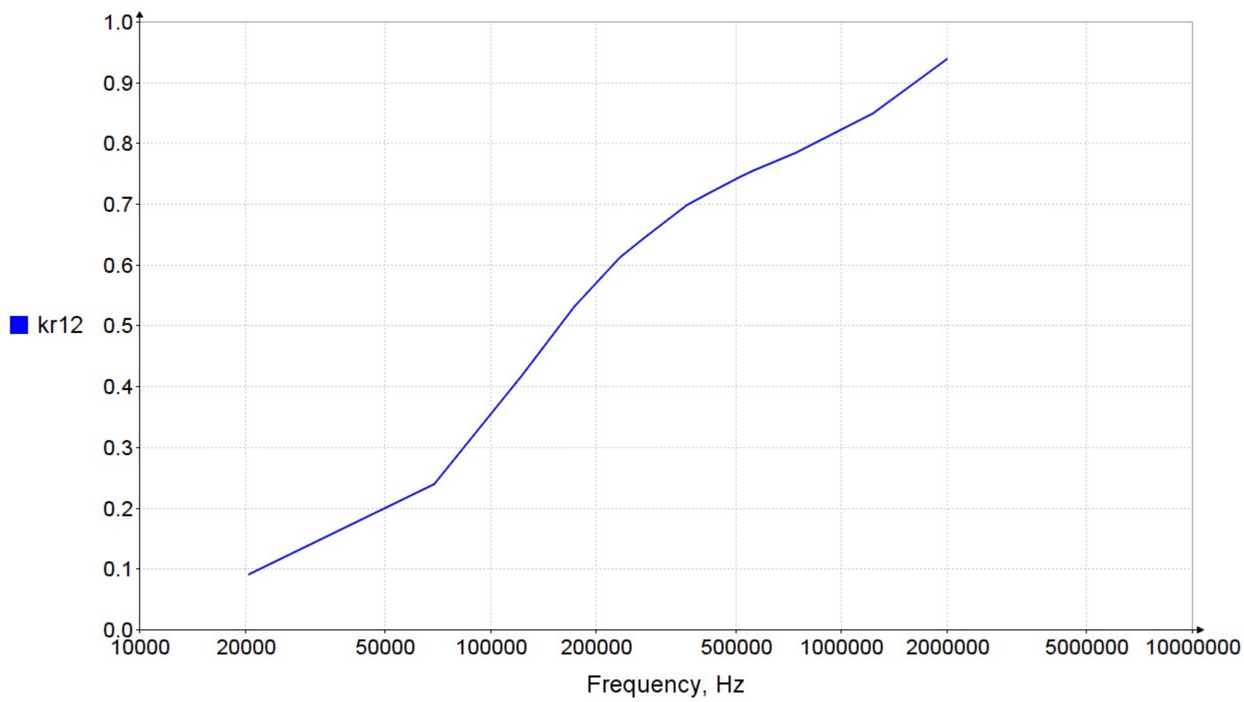
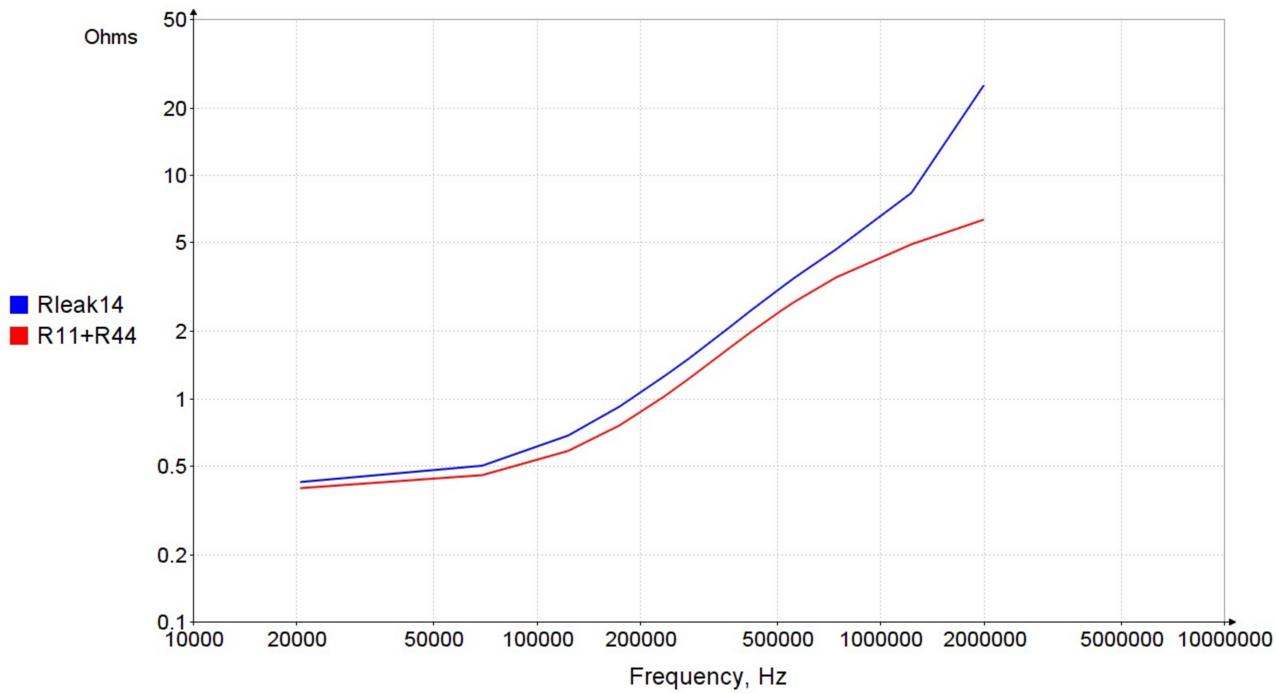


Fig. 10. Comparison between sum of self and reflected self resistances for high positive mutual resistance.

There is a significant reduction of the ac resistance due to the mutual resistance between these adjacent windings.

N1 = 18 Turns, N2 = 36 Turns, N3 = 36 Turns, N4 = 18 Turns



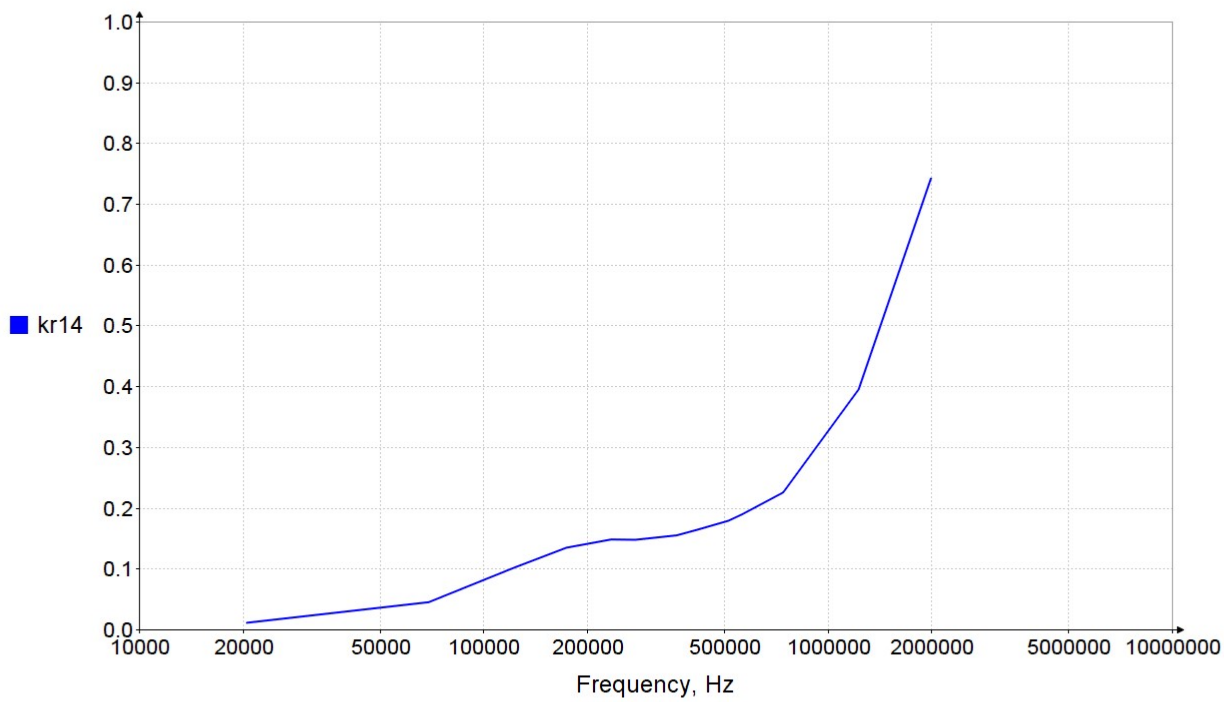


Fig. 11. Comparison between sum of self and reflected self resistances for lower mutual resistance.

These coils are not adjacent, and the resulting lower mutual resistance produces less reduction of the ac resistance compared to Fig. 10.

The impedance matrix description of the transformer can be approximated with the circuit model shown below in Fig. 12 as explained in [1]. For each winding, there is a resistor representing the dc resistance of that winding and a main inductor representing the maximum low-frequency inductance for that winding. The main inductor and the dc resistance for each winding are connected in series between the electrical terminals of that winding. There is also a set of auxiliary circuits for each winding that is shown in a row to the left of the winding. Each auxiliary circuit consists of an auxiliary inductor that is connected in parallel with an auxiliary resistor. The bottom terminals of all of the inductors in each set are connected together to prevent floating nodes, which are not allowed in circuit simulators.

The schematic diagram shows two auxiliary circuits for each winding, but the model could be extended to include more auxiliary circuits. Increasing the number of auxiliary circuits increases the frequency range over which the skin effect can be modeled.

The main inductors are coupled to each other and to each of the auxiliary inductors. The auxiliary inductors are not coupled to each other. It is, of course, impossible to construct a magnetic device in which a set of uncoupled windings are all coupled some other winding. This arrangement is useful as a model, however, and it is possible to describe it mathematically, and to model it in circuit simulators.

The model has one more degree of freedom than is necessary for each auxiliary inductors, so the inductances of the auxiliary inductors in each set are assigned a value equal to the inductance of the main winding associated with that set.

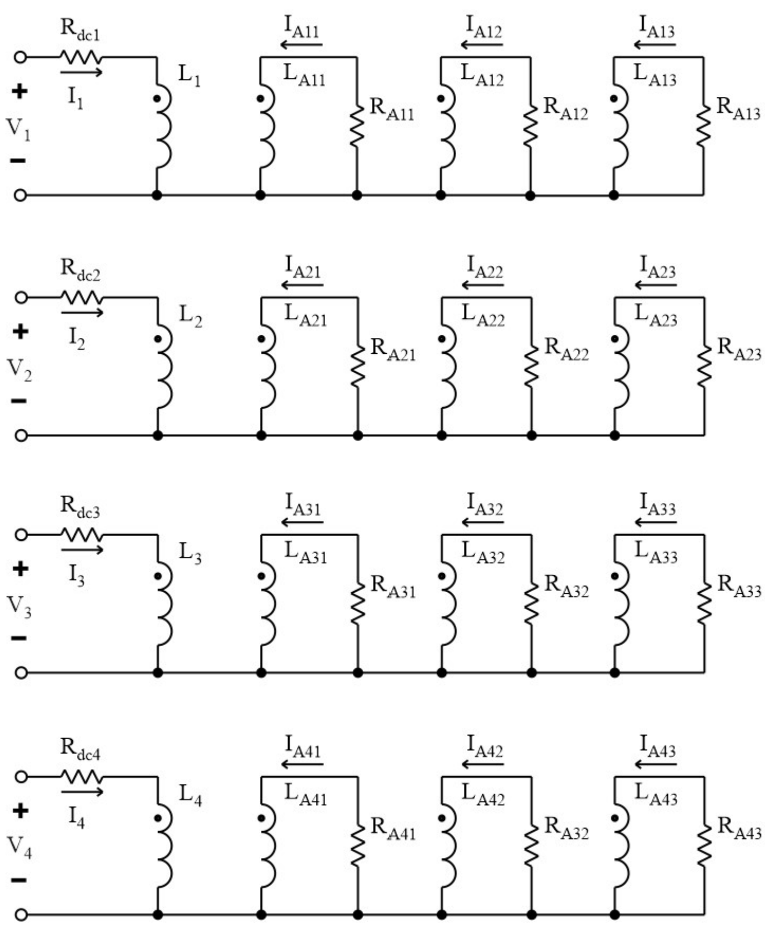


Figure 12. Schematic diagram of transformer circuit model.

We now define several variables and matrices that will be used in an equation that describes the circuit of Fig. 12.

The total number of auxiliary circuits for each winding is designated as r, which is 3 in Fig. 12. The counter variable κ indicates the κth auxiliary circuit, and it ranges from 1 to r.

$N := 4$ $\quad r := 3$ $\quad Naux := N \cdot r = 12$ $\quad Nk := N^2 \cdot r = 48$

The value of each auxiliary inductor is set equal to the value of the corresponding main winding.

$$L_A := \left\| \begin{array}{l} row \leftarrow 0 \\ \text{for } n \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } a \in 1..r \\ \quad \left\| \begin{array}{l} row \leftarrow row + 1 \\ L_{row} \leftarrow Lb_{n,n} \end{array} \right. \end{array} \right. \\ L \end{array} \right.$$

R_A contains the initial guess values of the auxiliary resistors for solve block.

k_A contains the initial guess values of the coupling coefficients between the auxiliary circuits and the main windings.

$$R_A := \begin{bmatrix} 1000 \\ 100 \\ 10 \\ 1000 \\ 100 \\ 10 \\ 1000 \\ 100 \\ 10 \\ 1000 \\ 100 \\ 10 \end{bmatrix} \cdot \Omega$$

$$k_A := \left\| \begin{array}{l} \text{for } n \in 1..Nk \\ \quad \left\| K_n \leftarrow 0.01 \right. \\ K \end{array} \right.$$

The mutual inductance between L_1 and L_2 is designated M_{12}.

The elements of M are calculated as shown below.

$$M\left(k_A\right) := \left\| \begin{array}{l} a \leftarrow 1 \\ Cols \leftarrow Naux \\ \text{for } row \in 1 \,..\, N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1 \,..\, Cols \\ \quad \left\| \begin{array}{l} M_{row\,,\,col} \leftarrow k_{A_a} \cdot \sqrt{L_{A_{col}} \cdot Lb_{row\,,\,row}} \\ a \leftarrow a + 1 \end{array} \right. \end{array} \right. \\ M \end{array} \right.$$

The value of each auxiliary inductor is set equal to the value of the corresponding main winding.

$$L_A := \left\| \begin{array}{l} row \leftarrow 0 \\ \text{for } n \in 1 \,..\, N \\ \quad \left\| \begin{array}{l} \text{for } a \in 1 \,..\, r \\ \quad \left\| \begin{array}{l} row \leftarrow row + 1 \\ L_{row} \leftarrow Lb_{n\,,\,n} \end{array} \right. \end{array} \right. \\ L \end{array} \right.$$

Functions to define the G and B matrices described in [1]. G and B are needed to find the impedances of the transformer equivalent circuit shown in Fig. 12.

$$G\left(f, L_A, R_A\right) := \left\| \begin{array}{l} G_A \leftarrow \overrightarrow{\left(\dfrac{R_A}{{R_A}^2 + \left(\omega_f\right)^2 \cdot {L_A}^2}\right)} \\ X \leftarrow \dfrac{\text{identity}\left(Naux\right)}{\boldsymbol{\Omega}} \\ \text{for } n \in 1 \,..\, Naux \\ \quad \left\| \ X_{n\,,\,n} \leftarrow G_{A_n} \right. \\ X \end{array} \right.$$

$$B\left(f, L_A, R_A\right) := \left\| \begin{array}{l} B_A \leftarrow \overrightarrow{\left(\dfrac{L_A}{{R_A}^2 + \left(\omega_f\right)^2 \cdot {L_A}^2}\right)} \\ X \leftarrow \text{identity}\left(Naux\right) \cdot \dfrac{\boldsymbol{s}}{\boldsymbol{\Omega}} \\ \text{for } n \in 1 \,..\, Naux \\ \quad \left\| \ X_{n\,,\,n} \leftarrow B_{A_n} \right. \\ X \end{array} \right.$$

$\text{rows}\left(G\left(1, L_A, R_A\right)\right) = 12$

$\text{cols}\left(G\left(1, L_A, R_A\right)\right) = 12$

$\text{rows}\left(B\left(1, L_A, R_A\right)\right) = 12$

$\text{cols}\left(B\left(1, L_A, R_A\right)\right) = 12$

Function to calculate the equivalent circuit self and mutual resistances.

$$R_{eq}\left(R_A, k_A, f\right) := \left\| \begin{array}{l} R \leftarrow Rb + \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot G\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \begin{bmatrix} R_{1,1} \\ R_{2,2} \\ R_{3,3} \\ R_{4,4} \\ R_{1,2} \\ R_{1,3} \\ R_{1,4} \\ R_{2,3} \\ R_{2,4} \\ R_{3,4} \end{bmatrix} \end{array} \right.$$

$$\mathrm{rows}\left(R_{eq}\left(R_A, k_A, 1\right)\right) = 10$$

$$\mathrm{cols}\left(R_{eq}\left(R_A, k_A, 1\right)\right) = 1$$

Function to calculate self and mutual inductances of the the equivalent circuit

$$L_{eq}\left(R_A, k_A, f\right) := \left\| \begin{array}{l} L \leftarrow Lb - \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot B\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \begin{bmatrix} L_{1,1} \\ L_{2,2} \\ L_{3,3} \\ L_{4,4} \\ L_{1,2} \\ L_{1,3} \\ L_{1,4} \\ L_{2,3} \\ L_{2,4} \\ L_{3,4} \end{bmatrix} \end{array} \right.$$

Function to calculate the equivalent circuit self and mutual impedances.

$$Z_{eq}\left(R_A, k_A, f\right) := R_{eq}\left(R_A, k_A, f\right) + 1\mathrm{j} \cdot \omega_f \cdot L_{eq}\left(R_A, k_A, f\right)$$

Functions to calculate leakage impedances based on (5).

$$Z_{leak\_mn} = Z_{mm} - \frac{Z_{mn}^{\;2}}{Z_{nn}} \qquad (5)$$

$$Rleak12EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_5\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_2}\right)$$

$$Rleak13EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_6\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_3}\right)$$

$$Rleak14EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_7\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

$$Rleak23EQ\left(R_A,k_A,f\right):=\mathrm{Re}\left(Z_{eq}\left(R_A,k_A,f\right)_2-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_8\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_3}\right)$$

$$Rleak24EQ\left(R_A,k_A,f\right):=\mathrm{Re}\left(Z_{eq}\left(R_A,k_A,f\right)_2-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_9\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_4}\right)$$

$$Rleak34EQ\left(R_A,k_A,f\right):=\mathrm{Re}\left(Z_{eq}\left(R_A,k_A,f\right)_3-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_{10}\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_4}\right)$$

$$Lleak12EQ\left(R_A,k_A,f\right):=\frac{1}{\omega_f}\cdot\mathrm{Im}\left(Z_{eq}\left(R_A,k_A,f\right)_1-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_5\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_2}\right)$$

$$Lleak13EQ\left(R_A,k_A,f\right):=\frac{1}{\omega_f}\cdot\mathrm{Im}\left(Z_{eq}\left(R_A,k_A,f\right)_1-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_6\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_3}\right)$$

$$Lleak14EQ\left(R_A,k_A,f\right):=\frac{1}{\omega_f}\cdot\mathrm{Im}\left(Z_{eq}\left(R_A,k_A,f\right)_1-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_7\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_4}\right)$$

$$Lleak23EQ\left(R_A,k_A,f\right):=\frac{1}{\omega_f}\cdot\mathrm{Im}\left(Z_{eq}\left(R_A,k_A,f\right)_2-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_8\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_3}\right)$$

$$Lleak24EQ\left(R_A,k_A,f\right):=\frac{1}{\omega_f}\cdot\mathrm{Im}\left(Z_{eq}\left(R_A,k_A,f\right)_2-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_9\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_4}\right)$$

$$Lleak34EQ\left(R_A,k_A,f\right):=\frac{1}{\omega_f}\cdot\mathrm{Im}\left(Z_{eq}\left(R_A,k_A,f\right)_3-\frac{\left(Z_{eq}\left(R_A,k_A,f\right)_{10}\right)^2}{Z_{eq}\left(R_A,k_A,f\right)_4}\right)$$

Function to calculate the mutual resistance couplings of the the equivalent circuit

$$kreq\left(R_A, k_A, f\right) := \left\| \begin{array}{l} R \leftarrow Rb + \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot G\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \begin{bmatrix} \dfrac{R_{1,2}}{\sqrt{R_{1,1} \cdot R_{2,2}}} \\ \dfrac{R_{1,3}}{\sqrt{R_{1,1} \cdot R_{3,3}}} \\ \dfrac{R_{1,4}}{\sqrt{R_{1,1} \cdot R_{4,4}}} \\ \dfrac{R_{2,3}}{\sqrt{R_{2,2} \cdot R_{3,3}}} \\ \dfrac{R_{2,4}}{\sqrt{R_{2,2} \cdot R_{4,4}}} \\ \dfrac{R_{3,4}}{\sqrt{R_{3,3} \cdot R_{4,4}}} \end{bmatrix} \end{array} \right.$$

Define error functions base on mutual resistance coupling, resistance and inductance

$$Error\_KR\left(R_A, k_A, f\right) := \left\| \begin{array}{l} kr \leftarrow KR(f) \\ kreq\left(R_A, k_A, f\right) - kr \end{array} \right.$$

$$Error\_R\left(R_A, k_A, f\right) := \left\| \begin{array}{l} Rf \leftarrow RF(f) \\ D \leftarrow R_{eq}\left(R_A, k_A, f\right) - Rf \\ \overrightarrow{\dfrac{D}{Rf}} \end{array} \right.$$

$$Error\_L\left(R_A, k_A, f\right) := \left\| \begin{array}{l} Lf \leftarrow LF(f) \\ D \leftarrow L_{eq}\left(R_A, k_A, f\right) - Lf \\ \overrightarrow{\dfrac{D}{Lf}} \end{array} \right.$$

Vectors of zeros used in the Minerr block.

$$Z := \left\| \begin{array}{l} Z_{\frac{N^2+N}{2}} \leftarrow 0 \\ Z \end{array} \right. \qquad Zk := \left\| \begin{array}{l} Z_{\frac{N^2-N}{2}} \leftarrow 0 \\ Z \end{array} \right.$$

Define a function for calculating the coupling matrix of the equivalent circuit.

$$K_{mod}(k_A) := \left\| \begin{array}{l} K \leftarrow \text{identity}(N+Naux) \\ \text{for } m \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } n \in 1..N \\ \quad \left\| K_{m,n} \leftarrow Kb_{m,n} \right. \end{array} \right. \\ a \leftarrow 1 \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in N+1..N+Naux \\ \quad \left\| \begin{array}{l} K_{row,col} \leftarrow k_{A_a} \\ K_{col,row} \leftarrow k_{A_a} \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ K \end{array} \right.$$

Define function for computing the eigenvalues of the model coupling matrix.

$$EigenTest(k_A) := \left\| \begin{array}{l} EV \leftarrow \text{eigenvals}(K_{mod}(k_A)) \\ EV \end{array} \right.$$

$$EigenTest(k_A) = \begin{bmatrix} 3.944 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0.998 \\ 0.042 \\ 0.011 \\ 0.005 \end{bmatrix}$$

Define function for determining the if there is a negative eigenvalue for the model coupling matrix. This helps the solver find solutions that are physically realizable, which requires all of the eigenvalues to be positive [2]. The 1000000 multiplier gives it a high weight in the solver.

$$EigenSign(k_A) := \left\| \begin{array}{l} ET \leftarrow min(EigenTest(k_A)) \\ \dfrac{10000000 \cdot ET}{|ET| + 10^{-10}} \end{array} \right. \qquad EigenSign(k_A) = 10 \cdot 10^{6}$$

Error Weighting $\quad c := 100 \quad d := 10 \quad e := 1$

$d \cdot Error\_R(R_A, k_A, 1) = Z \quad d \cdot Error\_R(R_A, k_A, 5) = Z \quad d \cdot Error\_R(R_A, k_A, 9) = Z$

$d \cdot Error\_R(R_A, k_A, 2) = Z \quad d \cdot Error\_R(R_A, k_A, 6) = Z \quad d \cdot Error\_R(R_A, k_A, 10) = Z$

$d \cdot Error\_R(R_A, k_A, 3) = Z \quad d \cdot Error\_R(R_A, k_A, 7) = Z \quad d \cdot Error\_R(R_A, k_A, 11) = Z$

$d \cdot Error\_R(R_A, k_A, 4) = Z \quad d \cdot Error\_R(R_A, k_A, 8) = Z \quad d \cdot Error\_R(R_A, k_A, 12) = Z$

$c \cdot Error\_L(R_A, k_A, 1) = Z \quad c \cdot Error\_L(R_A, k_A, 5) = Z \quad c \cdot Error\_L(R_A, k_A, 9) = Z$

$c \cdot Error\_L(R_A, k_A, 2) = Z \quad c \cdot Error\_L(R_A, k_A, 6) = Z \quad c \cdot Error\_L(R_A, k_A, 10) = Z$

$c \cdot Error\_L(R_A, k_A, 3) = Z \quad c \cdot Error\_L(R_A, k_A, 7) = Z \quad c \cdot Error\_L(R_A, k_A, 11) = Z$

$c \cdot Error\_L(R_A, k_A, 4) = Z \quad c \cdot Error\_L(R_A, k_A, 8) = Z \quad c \cdot Error\_L(R_A, k_A, 12) = Z$

Guess Values

$e \cdot Error\_KR(R_A, k_A, 1) = Zk \quad e \cdot Error\_KR(R_A, k_A, 5) = Zk \quad e \cdot Error\_KR(R_A, k_A, 9) = Zk$

$e \cdot Error\_KR(R_A, k_A, 2) = Zk \quad e \cdot Error\_KR(R_A, k_A, 6) = Zk \quad e \cdot Error\_KR(R_A, k_A, 10) = Zk$

$e \cdot Error\_KR(R_A, k_A, 3) = Zk \quad e \cdot Error\_KR(R_A, k_A, 7) = Zk \quad e \cdot Error\_KR(R_A, k_A, 11) = Zk$

$e \cdot Error\_KR(R_A, k_A, 4) = Zk \quad e \cdot Error\_KR(R_A, k_A, 8) = Zk \quad e \cdot Error\_KR(R_A, k_A, 12) = Zk$

traints

Constraints on coupling coefficients and resistors prevent inappropriate component values.

Cons

$-1 < k_{A_1} < 1$ $-1 < k_{A_9} < 1$ $-1 < k_{A_{17}} < 1$ $-1 < k_{A_{25}} < 1$ $-1 < k_{A_{33}} < 1$ $-1 < k_{A_{41}} < 1$

$-1 < k_{A_2} < 1$ $-1 < k_{A_{10}} < 1$ $-1 < k_{A_{18}} < 1$ $-1 < k_{A_{26}} < 1$ $-1 < k_{A_{34}} < 1$ $-1 < k_{A_{42}} < 1$

$-1 < k_{A_3} < 1$ $-1 < k_{A_{11}} < 1$ $-1 < k_{A_{19}} < 1$ $-1 < k_{A_{27}} < 1$ $-1 < k_{A_{35}} < 1$ $-1 < k_{A_{43}} < 1$

$-1 < k_{A_4} < 1$ $-1 < k_{A_{12}} < 1$ $-1 < k_{A_{20}} < 1$ $-1 < k_{A_{28}} < 1$ $-1 < k_{A_{36}} < 1$ $-1 < k_{A_{44}} < 1$

$-1 < k_{A_5} < 1$ $-1 < k_{A_{13}} < 1$ $-1 < k_{A_{21}} < 1$ $-1 < k_{A_{29}} < 1$ $-1 < k_{A_{37}} < 1$ $-1 < k_{A_{45}} < 1$

$-1 < k_{A_6} < 1$ $-1 < k_{A_{14}} < 1$ $-1 < k_{A_{22}} < 1$ $-1 < k_{A_{30}} < 1$ $-1 < k_{A_{38}} < 1$ $-1 < k_{A_{46}} < 1$

$-1 < k_{A_7} < 1$ $-1 < k_{A_{15}} < 1$ $-1 < k_{A_{23}} < 1$ $-1 < k_{A_{31}} < 1$ $-1 < k_{A_{39}} < 1$ $-1 < k_{A_{47}} < 1$

$-1 < k_{A_8} < 1$ $-1 < k_{A_{16}} < 1$ $-1 < k_{A_{24}} < 1$ $-1 < k_{A_{32}} < 1$ $-1 < k_{A_{40}} < 1$ $-1 < k_{A_{48}} < 1$

$10^4 \cdot R_{A_1} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_4} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_7} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_{10}} > 10^5 \cdot \Omega$

$10^4 \cdot R_{A_2} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_5} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_8} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_{11}} > 10^5 \cdot \Omega$

$10^4 \cdot R_{A_3} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_6} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_9} > 10^5 \cdot \Omega$ $10^4 \cdot R_{A_{12}} > 10^5 \cdot \Omega$

$EigenSign\left(k_A\right) > 10000000$

Solver

$$\begin{bmatrix} R_A \\ k_A \end{bmatrix} := \mathbf{Minerr}\left(R_A, k_A\right)$$

$ERR = ?$

$$R_A = \begin{bmatrix} 783.9398 \\ 981.8784 \\ 69.7503 \\ 3.9901 \cdot 10^3 \\ 710.7659 \\ 62.8531 \\ 4.0219 \cdot 10^3 \\ 683.0379 \\ 10.0000 \\ 820.7976 \\ 992.9180 \\ 261.8718 \end{bmatrix} \Omega$$

$$Kb = \begin{bmatrix} 1.00000 & 0.99163 & 0.97861 & 0.96349 \\ 0.99163 & 1.00000 & 0.98935 & 0.97402 \\ 0.97861 & 0.98935 & 1.00000 & 0.98715 \\ 0.96349 & 0.97402 & 0.98715 & 1.00000 \end{bmatrix}$$

$$\text{eigenvals}\left(K_{mod}\left(k_A\right)\right) = \begin{bmatrix} 3.978772 \\ 1.020776 \\ 1.007422 \\ 1.001277 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 0.964762 \\ 0.021448 \\ 0.003923 \\ 0.001620 \end{bmatrix}$$

If negative eigenvalues values are present then the realizability criterion is violated [2].

Computed auxiliary couplings

$$k_{A1} := \left\| \begin{array}{l} \text{for } h \in 1..24 \\ \quad \left\| K_h \leftarrow k_{A_h} \right. \\ K \end{array} \right.$$

$$k_{A2} := \left\| \begin{array}{l} \text{for } h \in 25..48 \\ \quad \left\| K_{h-24} \leftarrow k_{A_h} \right. \\ K \end{array} \right.$$

$$k_{A1} = \begin{bmatrix} 0.069179 \\ 0.041484 \\ 0.062593 \\ 0.062957 \\ -0.017083 \\ -0.007143 \\ 0.062025 \\ -0.015824 \\ -0.000097 \\ 0.048061 \\ 0.038979 \\ 0.067791 \\ 0.079815 \\ 0.052201 \\ 0.079492 \\ 0.065831 \\ 0.001333 \\ 0.019805 \\ 0.065779 \\ 0.054106 \\ 0.011679 \\ 0.077916 \\ 0.050492 \\ 0.060360 \end{bmatrix}$$

$$k_{A2} = \begin{bmatrix} 0.061637 \\ 0.049381 \\ 0.038670 \\ 0.062068 \\ 0.038466 \\ 0.016451 \\ 0.062131 \\ 0.076614 \\ 0.059818 \\ 0.071115 \\ 0.047870 \\ -0.015735 \\ 0.022667 \\ 0.068666 \\ 0.042355 \\ 0.061238 \\ 0.112004 \\ 0.032624 \\ 0.061227 \\ 0.058325 \\ -0.025515 \\ 0.034682 \\ 0.067596 \\ -0.010353 \end{bmatrix}$$

Extract resistance and inductance values of the equivalent circuit for plotting.

$R11eq_f := R_{eq}(R_A, k_A, f)_1$ $L11eq_f := L_{eq}(R_A, k_A, f)_1$

$R22eq_f := R_{eq}(R_A, k_A, f)_2$ $L22eq_f := L_{eq}(R_A, k_A, f)_2$

$R33eq_f := R_{eq}(R_A, k_A, f)_3$ $L33eq_f := L_{eq}(R_A, k_A, f)_3$

$R44eq_f := R_{eq}(R_A, k_A, f)_4$ $L44eq_f := L_{eq}(R_A, k_A, f)_4$

$R12eq_f := R_{eq}(R_A, k_A, f)_5$ $L12eq_f := L_{eq}(R_A, k_A, f)_5$

$R13eq_f := R_{eq}(R_A, k_A, f)_6$ $L13eq_f := L_{eq}(R_A, k_A, f)_6$

$R14eq_f := R_{eq}(R_A, k_A, f)_7$ $L14eq_f := L_{eq}(R_A, k_A, f)_7$

$R23eq_f := R_{eq}(R_A, k_A, f)_8$ $L23eq_f := L_{eq}(R_A, k_A, f)_8$

$R24eq_f := R_{eq}(R_A, k_A, f)_9$ $L24eq_f := L_{eq}(R_A, k_A, f)_9$

$R34eq_f := R_{eq}(R_A, k_A, f)_{10}$ $L34eq_f := L_{eq}(R_A, k_A, f)_{10}$

Equivalent Circuit mutual resistance couplings

$kr12eq_f := kreq(R_A, k_A, f)_1$ $kr13eq_f := kreq(R_A, k_A, f)_2$ $kr14eq_f := kreq(R_A, k_A, f)_3$

$kr23eq_f := kreq(R_A, k_A, f)_4$ $kr24eq_f := kreq(R_A, k_A, f)_5$ $kr34eq_f := kreq(R_A, k_A, f)_6$

Equivalent circuit leakage resistances and inductances

$$Rleak12eq_f := Rleak12EQ\left(R_A, k_A, f\right)$$

$$Rleak13eq_f := Rleak13EQ\left(R_A, k_A, f\right)$$

$$Rleak14eq_f := Rleak14EQ\left(R_A, k_A, f\right)$$

$$Rleak23eq_f := Rleak23EQ\left(R_A, k_A, f\right)$$

$$Rleak24eq_f := Rleak24EQ\left(R_A, k_A, f\right)$$

$$Rleak34eq_f := Rleak34EQ\left(R_A, k_A, f\right)$$

$$Lleak12eq_f := Lleak12EQ\left(R_A, k_A, f\right)$$

$$Lleak13eq_f := Lleak13EQ\left(R_A, k_A, f\right)$$

$$Lleak14eq_f := Lleak14EQ\left(R_A, k_A, f\right)$$

$$Lleak23eq_f := Lleak23EQ\left(R_A, k_A, f\right)$$

$$Lleak24eq_f := Lleak24EQ\left(R_A, k_A, f\right)$$

$$Lleak34eq_f := Lleak34EQ\left(R_A, k_A, f\right)$$

$$Qleak12_f := \frac{\omega_f \cdot Lleak12_f}{Rleak12_f}$$

$$Qleak13_f := \frac{\omega_f \cdot Lleak13_f}{Rleak13_f}$$

$$Qleak14_f := \frac{\omega_f \cdot Lleak14_f}{Rleak14_f}$$

$$Qleak23_f := \frac{\omega_f \cdot Lleak23_f}{Rleak23_f}$$

$$Qleak24_f := \frac{\omega_f \cdot Lleak24_f}{Rleak24eq_f}$$

$$Qleak34_f := \frac{\omega_f \cdot Lleak34_f}{Rleak34_f}$$

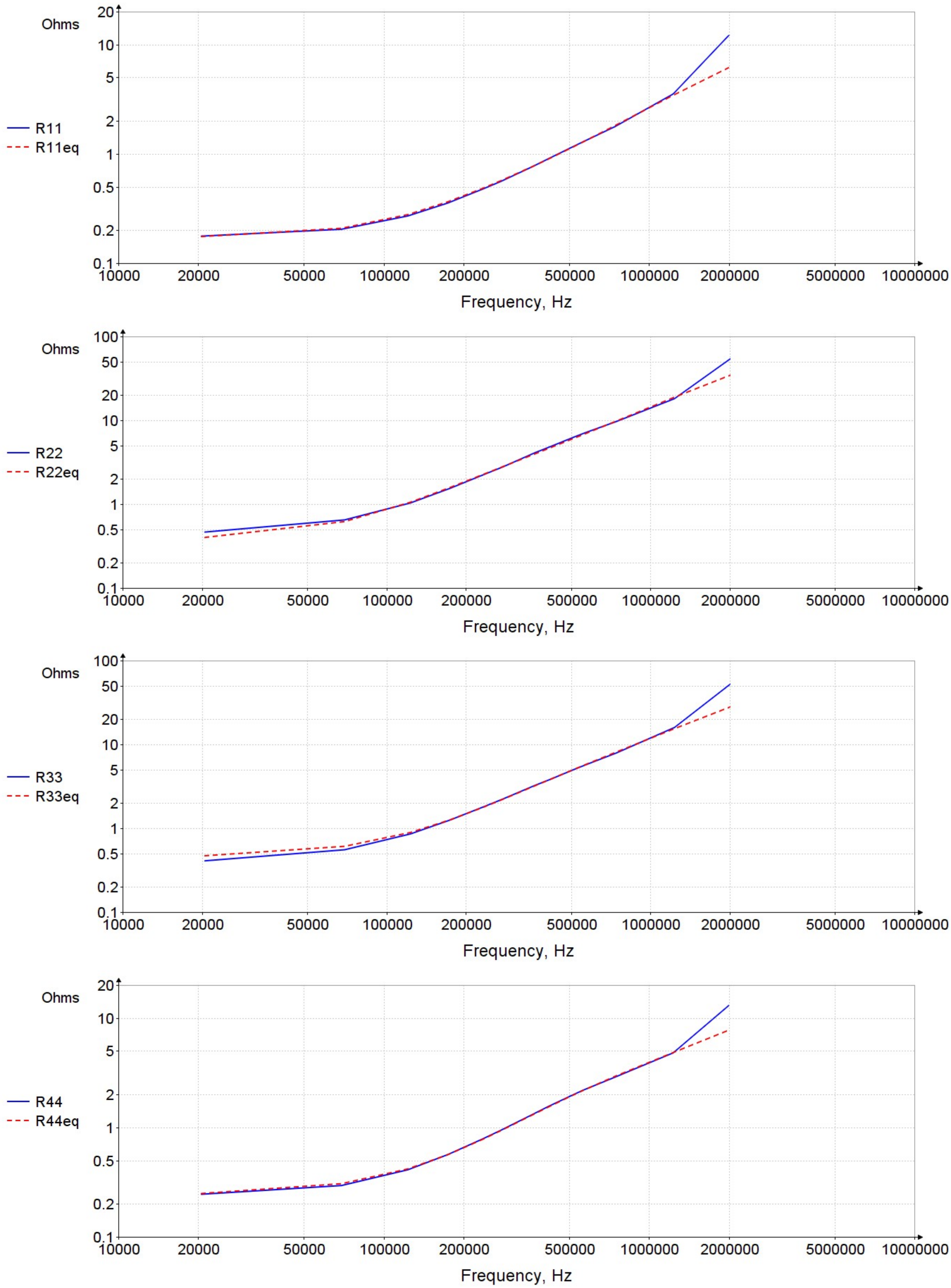
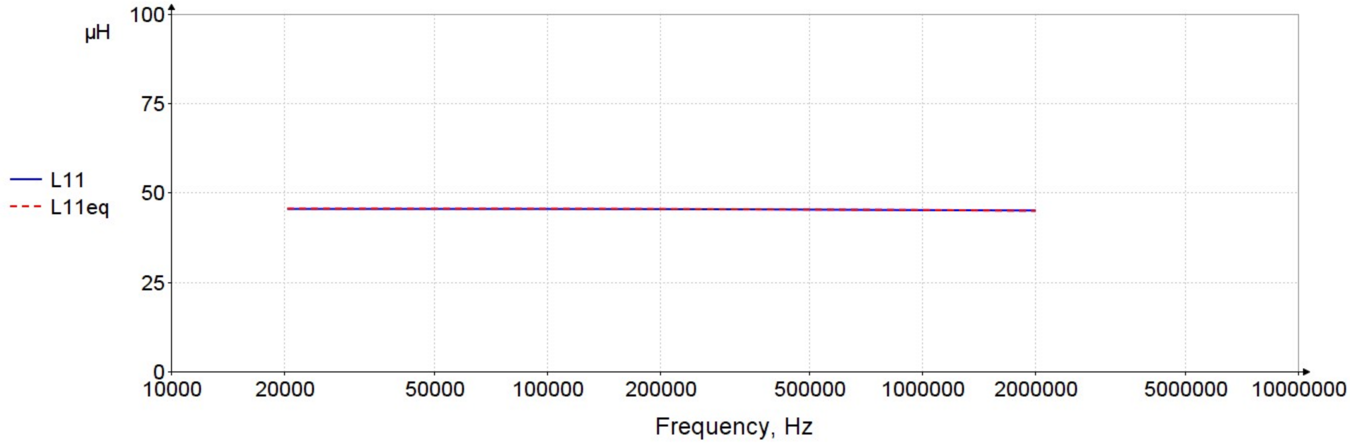
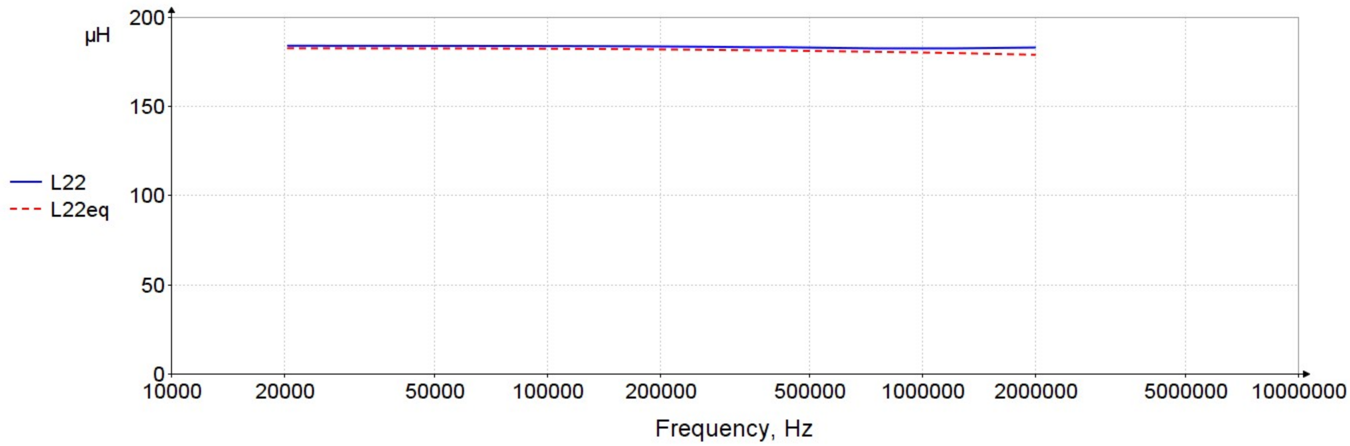
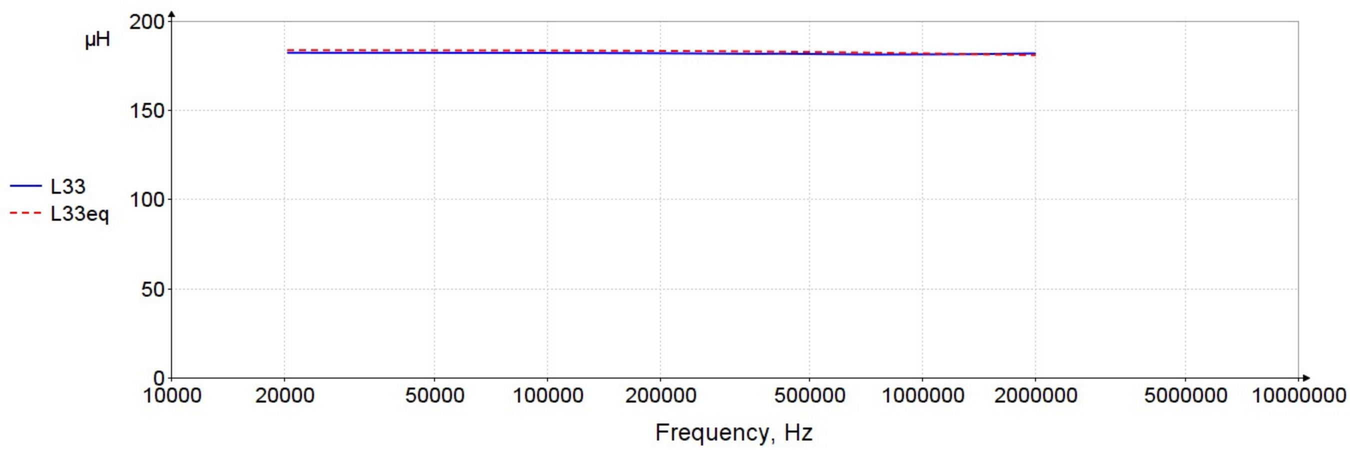


Figure 13. Measured and Equivalent Circuit self resistances.







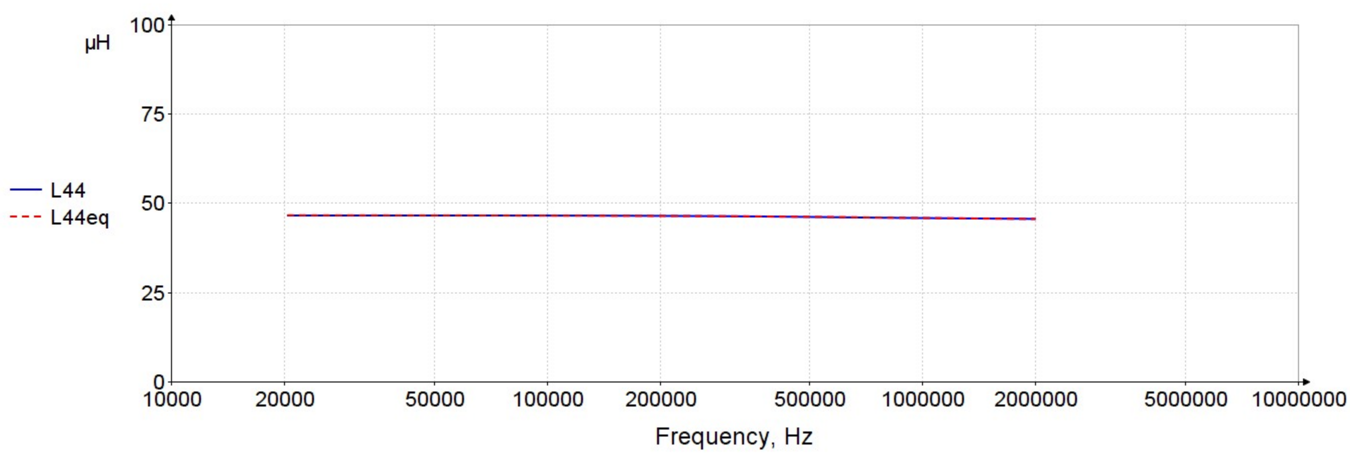


Figure 14. Measured and Equivalent Circuit self inductances.

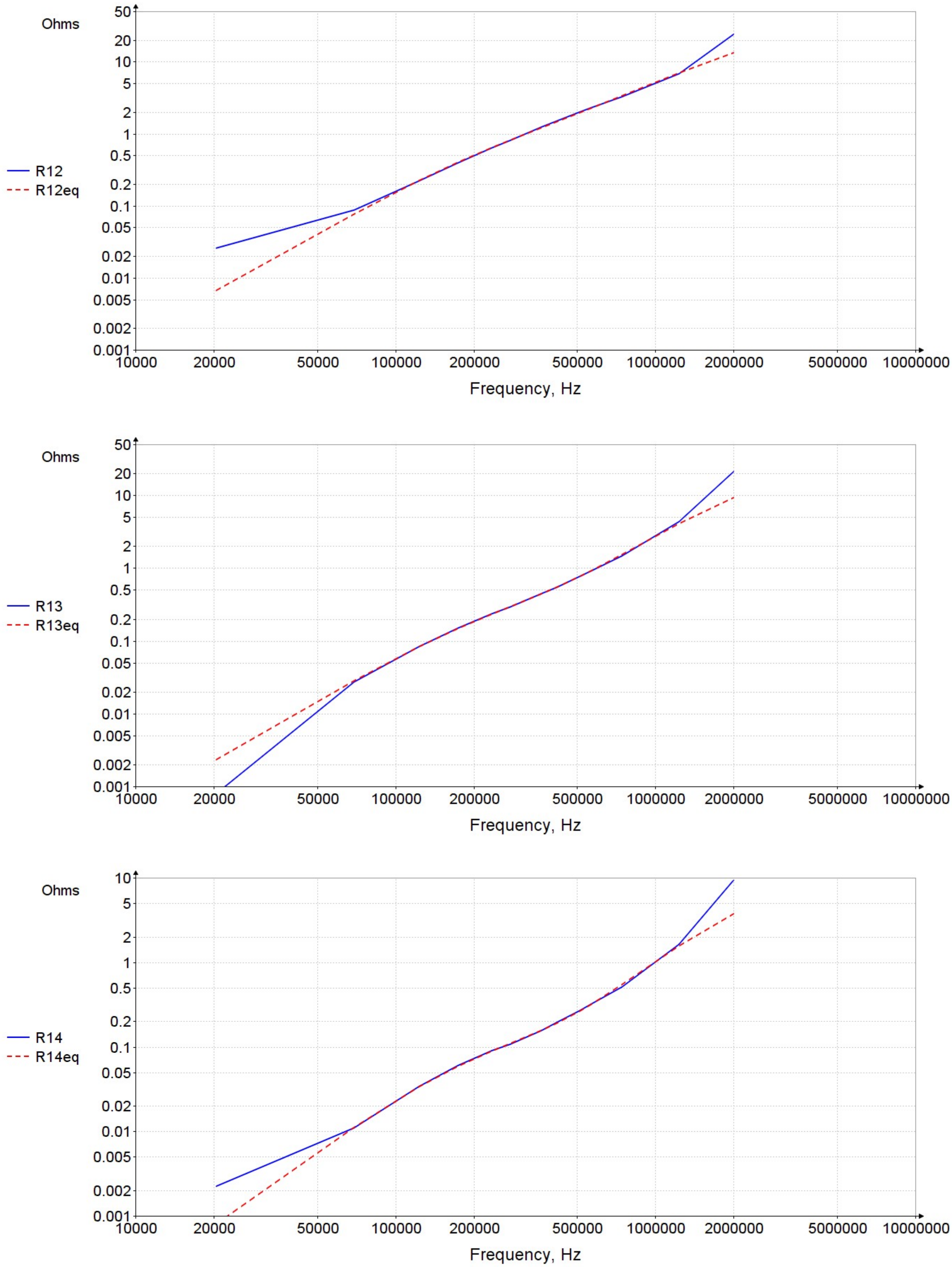


Figure 15a. Measured and Equivalent Circuit mutual resistances.

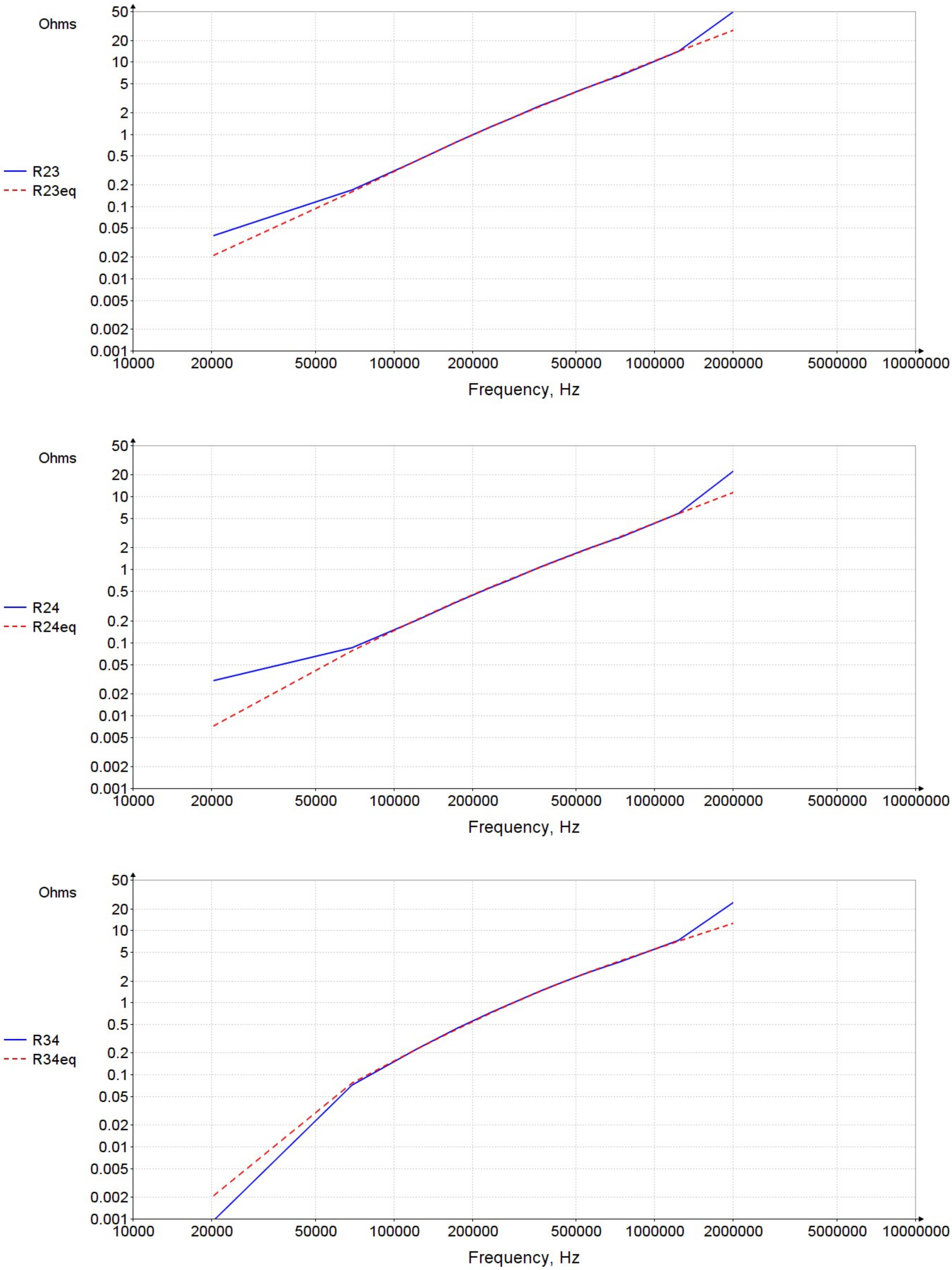


Figure 15b. Measured and Equivalent Circuit mutual resistances.

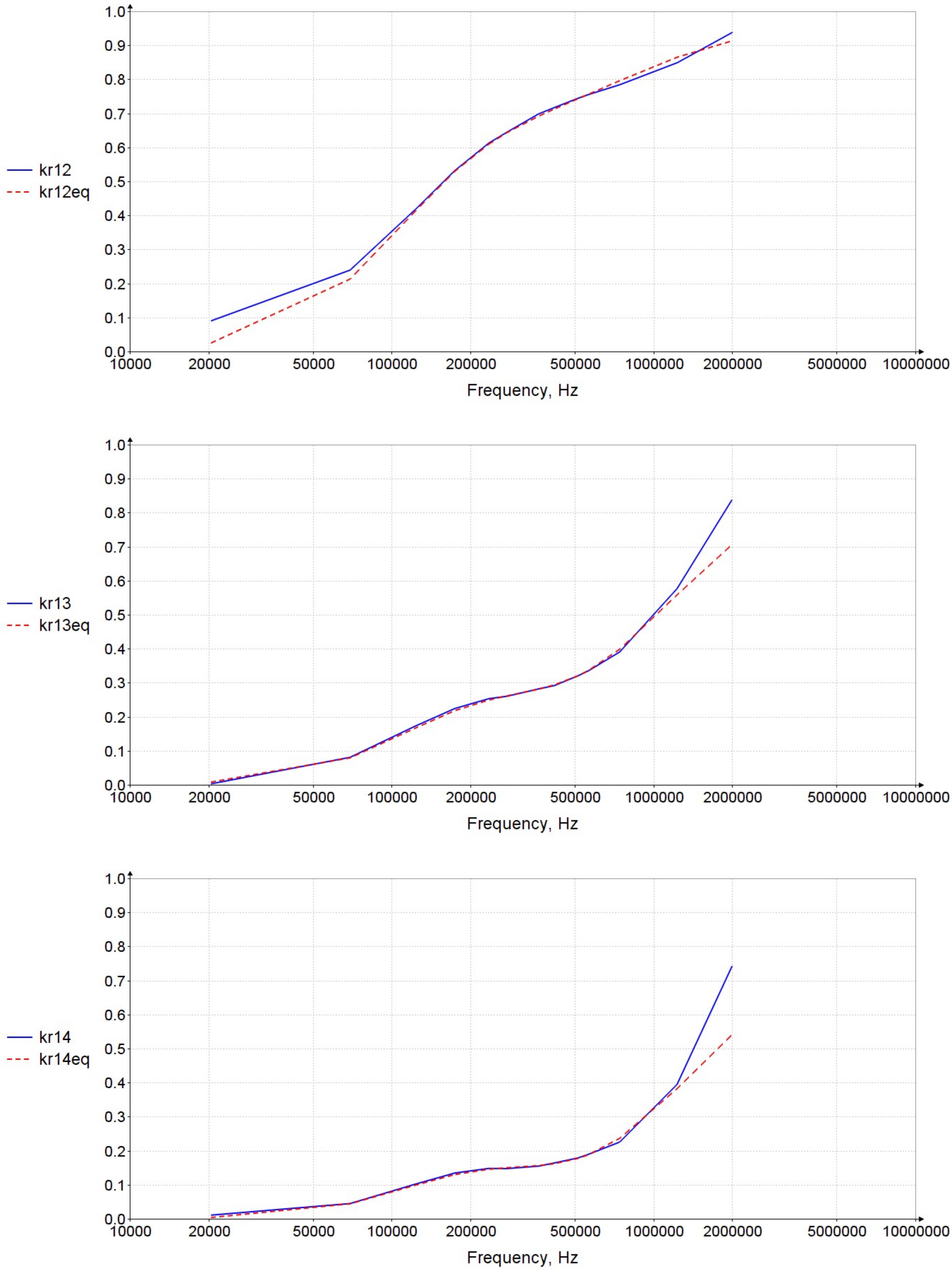
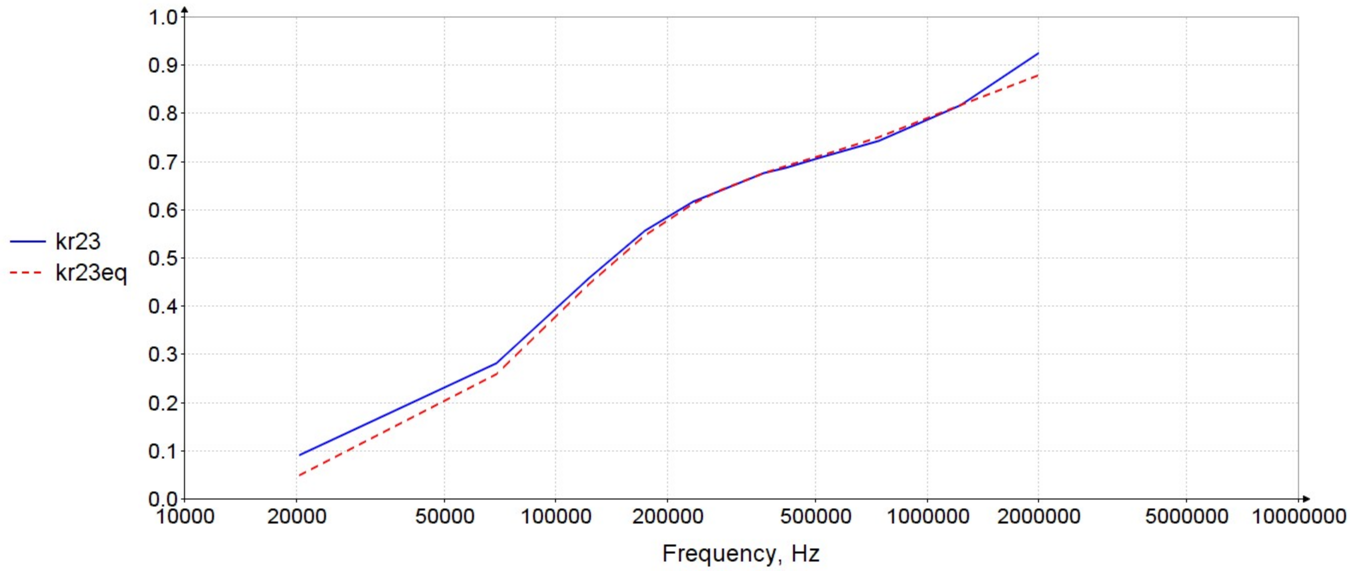
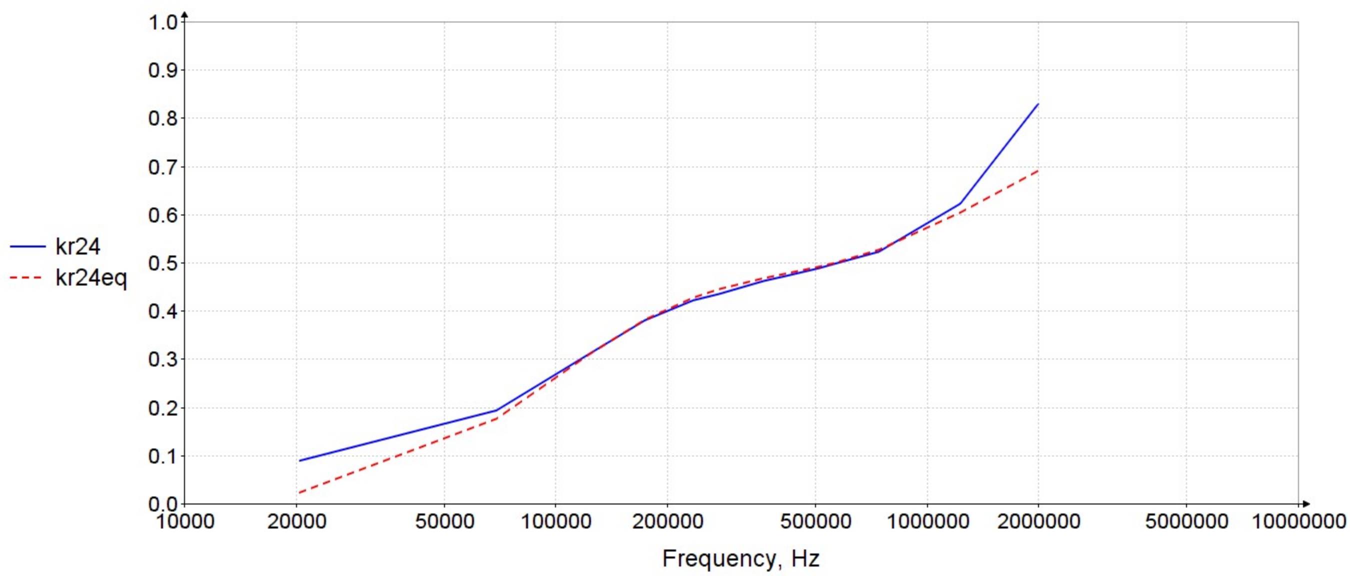


Figure 16a. Measured and Equivalent Circuit mutual resistance couplings.





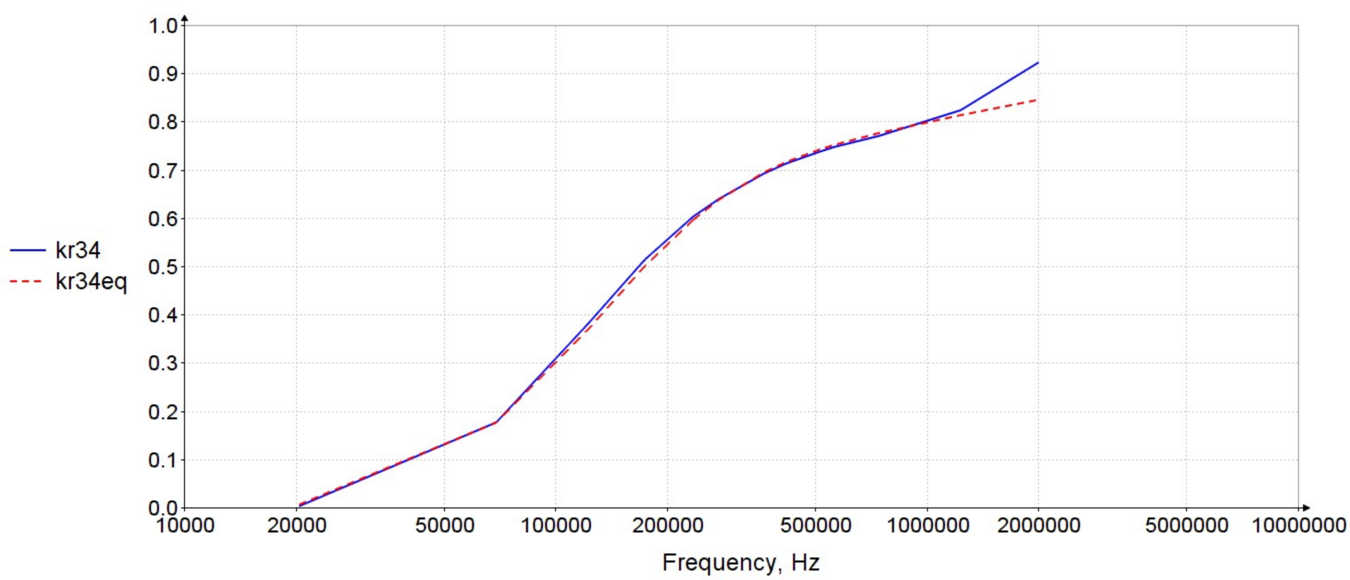


Figure 16b. Measured and Equivalent Circuit mutual resistance couplings.

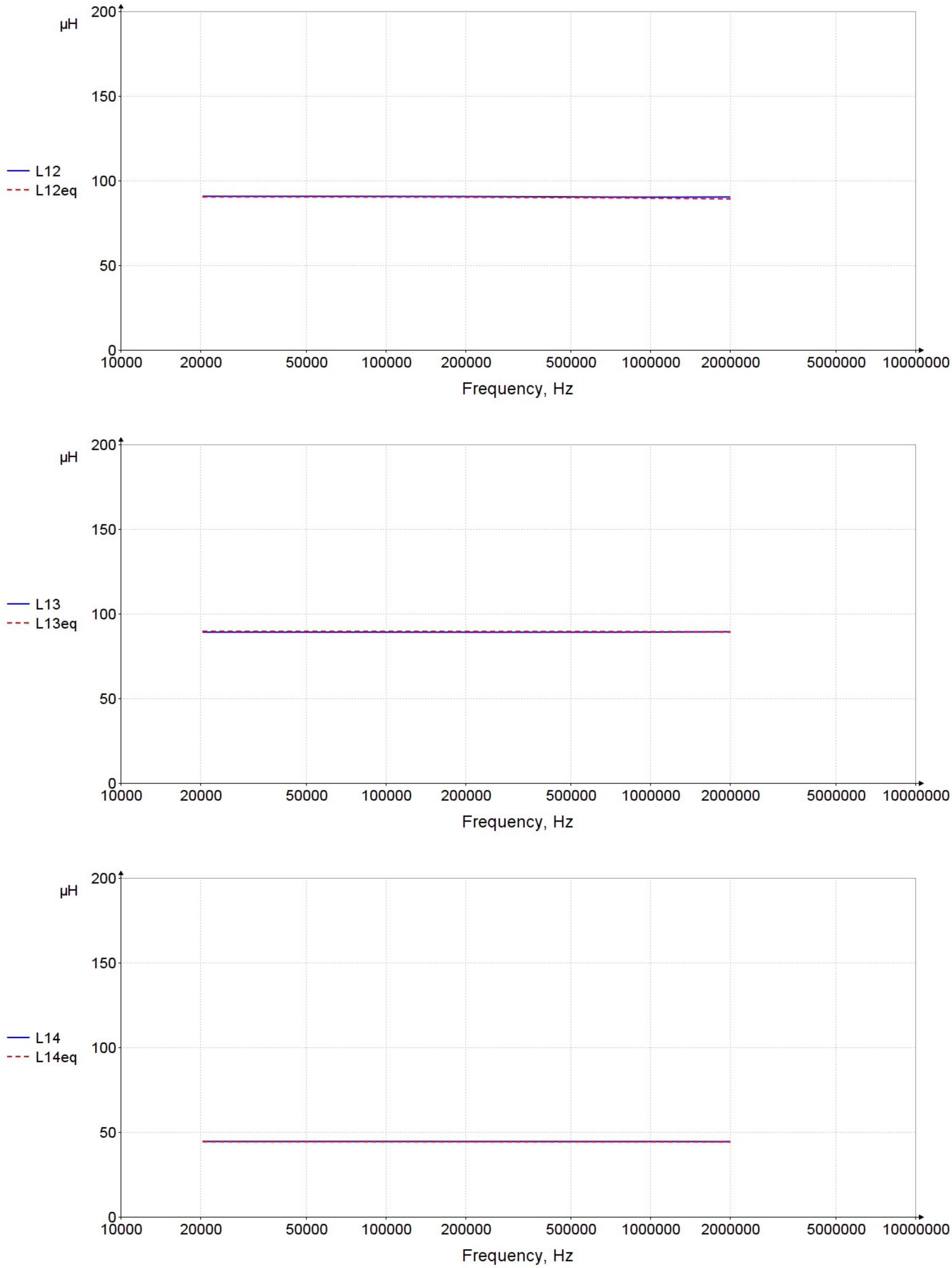


Figure 17a. Measured and Equivalent Circuit mutual inductances.

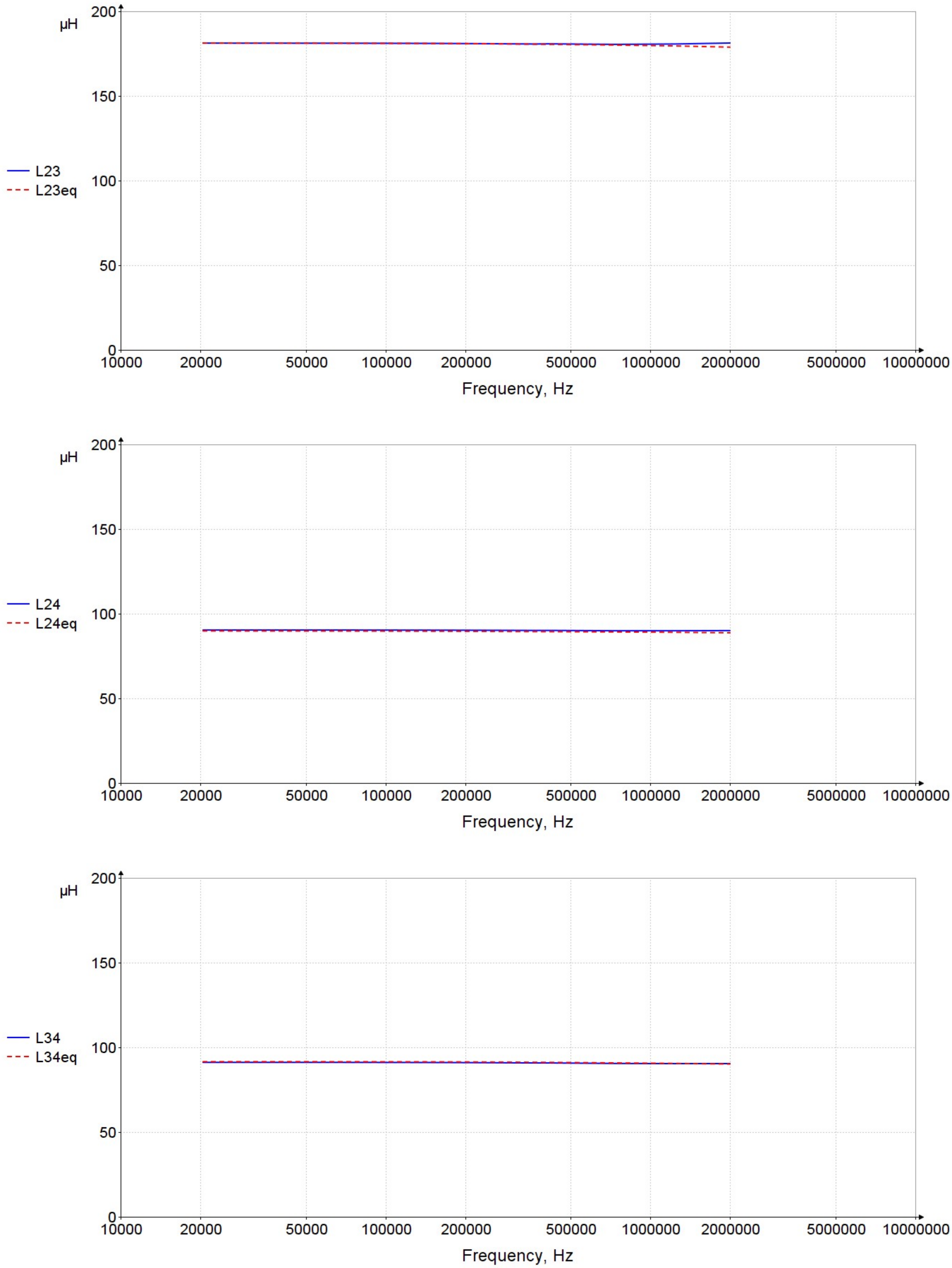


Figure 17b. Measured and Equivalent Circuit mutual inductances.

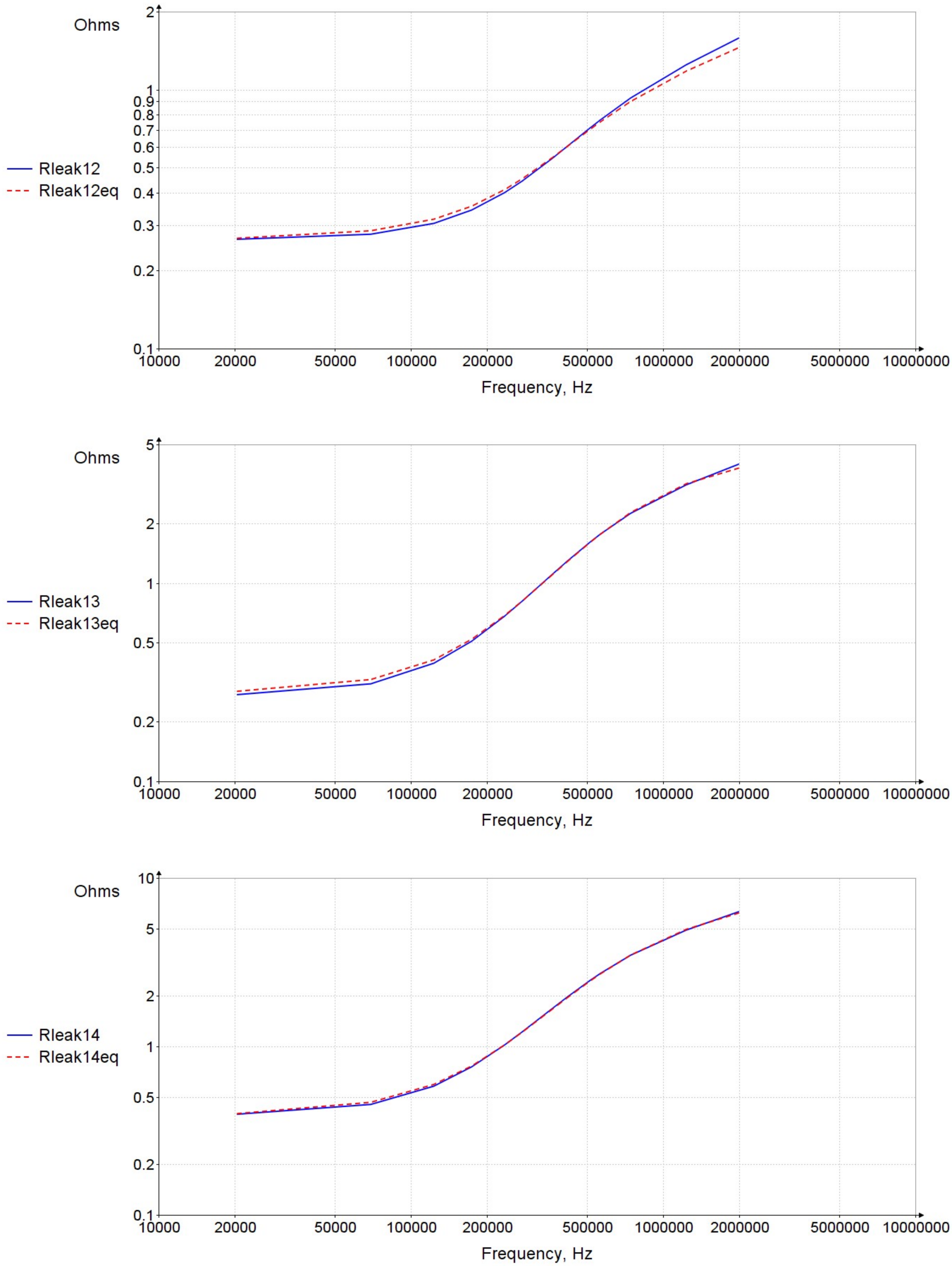


Fig. 18a. Measured and Equivalent Circuit leakage resistances.

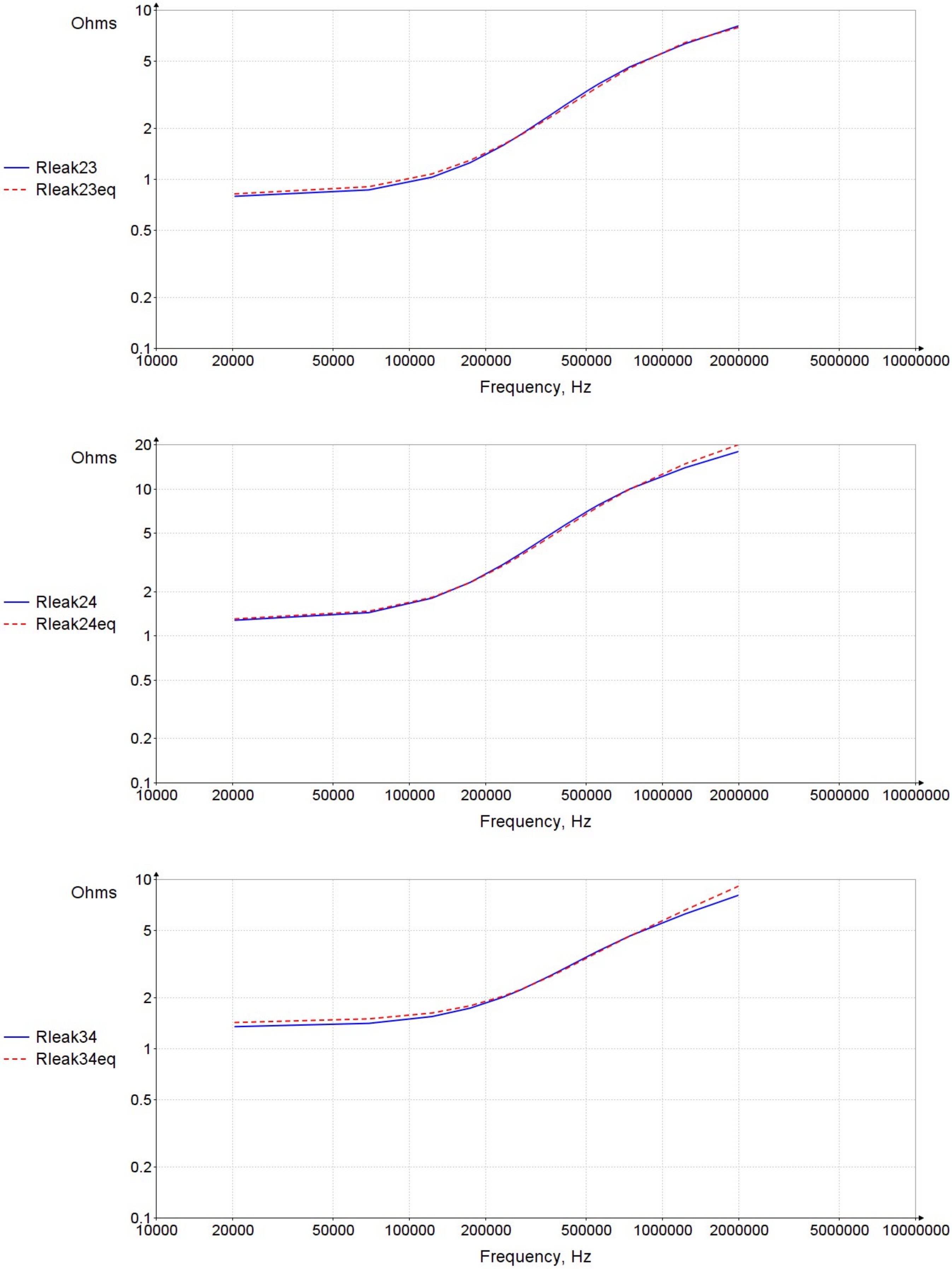


Fig. 18b. Measured and Equivalent Circuit leakage resistances.

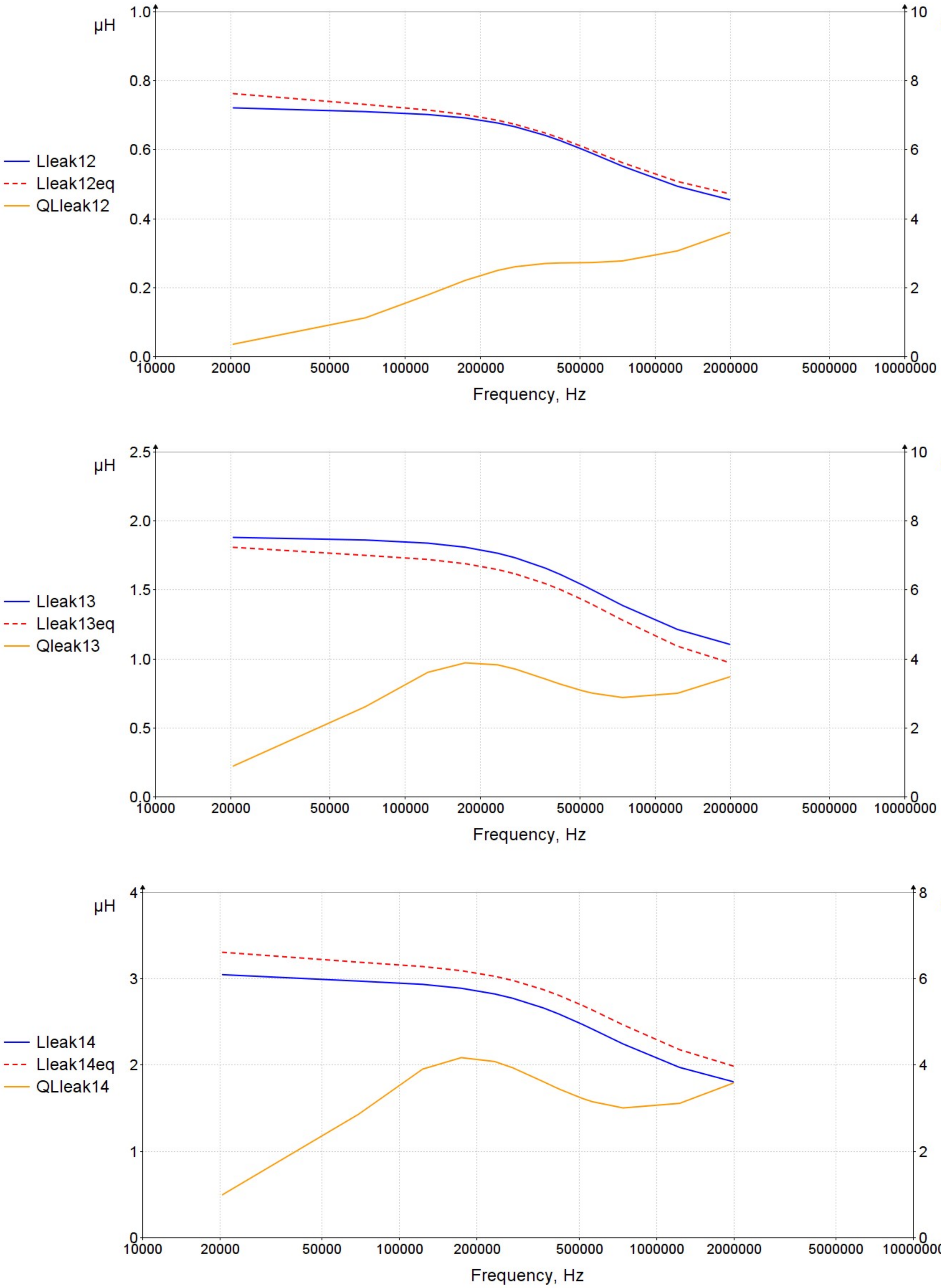


Fig. 19a. Measured and Equivalent Circuit leakage inductances.

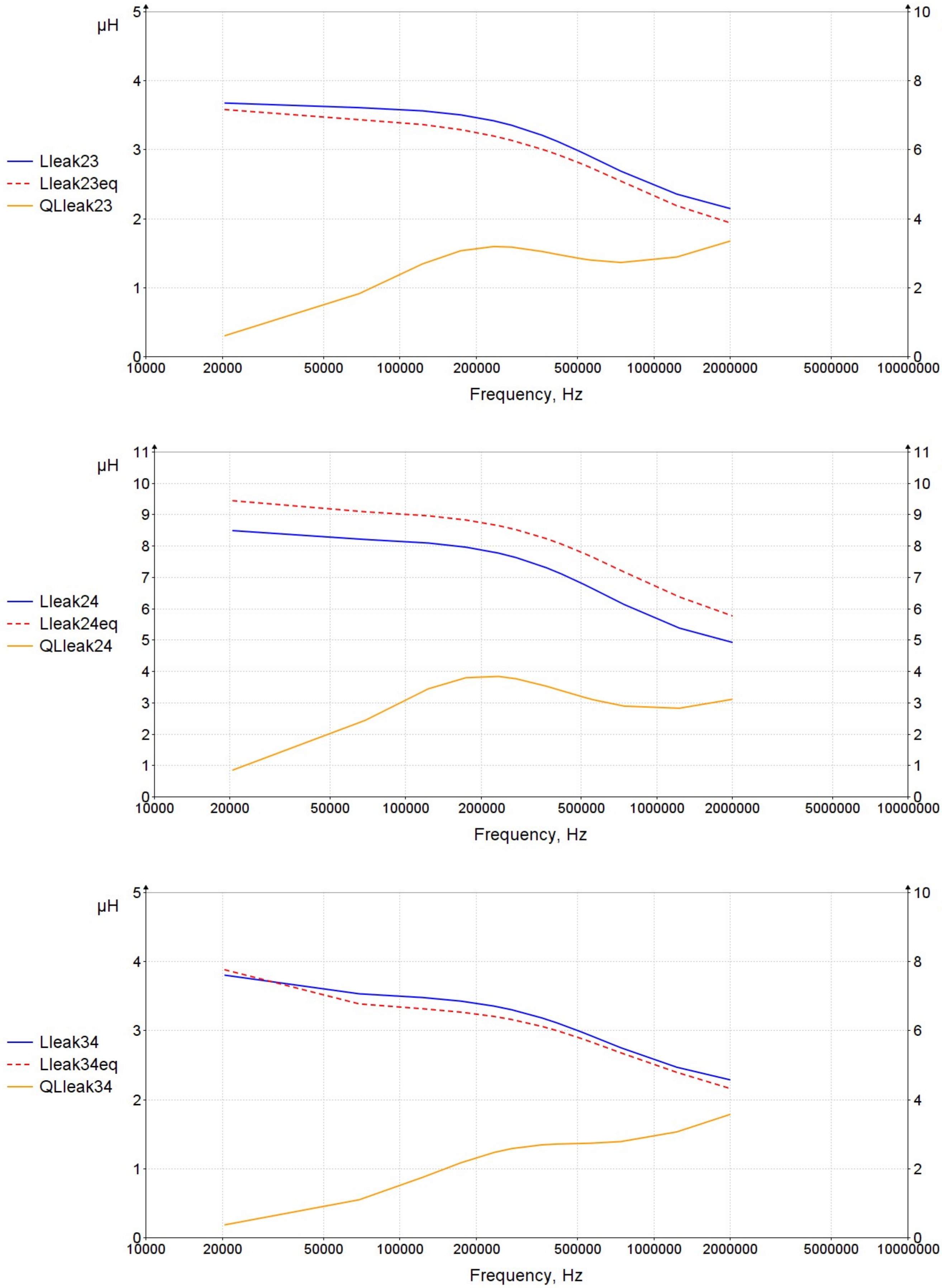


Fig. 19b. Measured and Equivalent Circuit leakage inductances.

References

[1] E. E. Mombello and K. Moller, "New power transformer model for the calculation of electromagnetic resonant transient phenomena including frequency-dependent losses," IEEE Transactions on Power Delivery, vol. 15, No. 1, January 2000, pp. 167-174.
http://ieeexplore.ieee.org/document/847246/

[2] B. L. Hesterman, E. E. Mombello and K. Moller, "Discussion of "New power transformer model for the calculation of electromagnetic resonant transient phenomena including frequency-dependent losses" [Closure to discussion]," IEEE Transactions on Power Delivery, vol. 15, No. no. 4, pp. 1320-1323, Oct. 2000
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[3] Yilmaz Tokad and Myril B. Reed, “Criteria and Tests for Realizability of the Inductance Matrix," Trans. AIEE, Part I, Communications and Electronics, Vol. 78, Jan. 1960, pp. 924-926
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[4] James Spreen, "Electrical terminal representation of conductor loss in transformers," IEEE Transactions on Power Electronics, vol. 5, No. 4, Oct 1990, pp. 424-429.
http://ieeexplore.ieee.org/document/60685/

Calculate strings for exporting the Lb and Rb values to LTspice.

$$paramLB := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{".param "} \\ \text{for } m \in 2..\text{rows}(Lb) \\ \quad \left\| STR_{m,1} \leftarrow \text{"+ } \quad \text{"} \right. \\ STR \end{array} \right.$$

$$LBnum := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Lb) \\ \quad \left\| STR_{m} \leftarrow \text{concat}(\text{"Lb"}, \text{num2str}(m), \text{"="}) \right. \\ STR \end{array} \right.$$

$$LB := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Lb) \\ \quad \left\| STR_{m} \leftarrow \text{concat}\left(paramLB_{m}, LBnum_{m}, \text{num2str}\left(\frac{Lb_{m,m}}{\boldsymbol{H}}\right)\right) \right. \\ STR \end{array} \right.$$

$$paramRB := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{".param "} \\ \text{for } m \in 2..\text{rows}(Rb) \\ \quad \left\| STR_{m,1} \leftarrow \text{"+ } \quad \text{"} \right. \\ STR \end{array} \right.$$

$$RBnum := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Rb) \\ \quad \left\| STR_{m} \leftarrow \text{concat}(\text{"Rb"}, \text{num2str}(m), \text{"="}) \right. \\ STR \end{array} \right.$$

$$RB := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Rb) \\ \quad \left\| STR_{m} \leftarrow \text{concat}\left(paramRB_{m}, RBnum_{m}, \text{num2str}\left(\frac{Rb_{m,m}}{\boldsymbol{\Omega}}\right)\right) \right. \\ STR \end{array} \right.$$

Calculate strings for exporting the RA, LA and KA values.

$$paramRA := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{".param "} \\ \text{for } m \in 2..\text{rows}(R_A) \\ \quad \| \; STR_{m,1} \leftarrow \text{"+ \quad "} \\ STR \end{array} \right.$$

$$RAnum := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } m \in 1..N \\ \quad \| \; \text{for } n \in 1..r \\ \quad \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}(\text{"RA"}, \text{num2str}(m), \text{num2str}(n), \text{"="}) \\ a \leftarrow a + 1 \end{array} \right. \\ STR \end{array} \right.$$

$$RA := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(R_A) \\ \quad \| \; STR_m \leftarrow \text{concat}\left(paramRA_m, RAnum_m, \text{num2str}\left(\dfrac{R_{A_m}}{\boldsymbol{\Omega}}\right)\right) \\ STR \end{array} \right.$$

$$LAnum := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } row \in 1..N \\ \quad \| \; \text{for } col \in 1..N \\ \quad \quad \| \; \text{for } \kappa \in 1..r \\ \quad \quad \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}(\text{num2str}(col), \text{num2str}(\kappa)) \\ a \leftarrow a + 1 \end{array} \right. \\ STR \end{array} \right.$$

$$KA := \left\| \begin{array}{l} a \leftarrow 1 \\ Cols \leftarrow \text{rows}(L_A) \\ \text{for } row \in 1..N \\ \quad \| \; \text{for } col \in 1..Cols \\ \quad \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}\left(\text{"KA"}, \text{num2str}(a), \text{" Lb"}, \text{num2str}(row), \text{" LA"}, LAnum_a, \text{" "}, \text{num2str}\left(k_{A_a}\right)\right) \\ a \leftarrow a + 1 \end{array} \right. \\ STR \end{array} \right.$$

Calculate string for exporting the Kb values.

$$
KB := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } row \in 1 .. N \\ \quad \left\| \begin{array}{l} \text{for } col \in row .. N \\ \quad \left\| \begin{array}{l} \text{if } row \neq col \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}\left(\text{“Kb”}, \text{num2str}(a), \text{“ Lb”}, \text{num2str}(row), \text{“ Lb”}, \text{num2str}(col), \text{“ ”}, \text{num2str}\left(Kb_{row, col}\right)\right) \\ a \leftarrow a + 1 \end{array} \right. \end{array} \right. \end{array} \right. \\ STR \end{array} \right.
$$

Combine the strings of model parameters for exporting.

$XFMR\_Params := \text{stack}(LB, RB, RA, KA, KB)$

Convert each string to its binary representation using str2vec, and add a CR=13 and LF=10 at the end of each string.

$\mathbf{ORIGIN} = 1$

$rowCount := \text{rows}(XFMR\_Params) = 74$

$indices := \mathbf{ORIGIN} .. (rowCount - 1 + \mathbf{ORIGIN})$

$$
XFMR\_Bin := \left\| \begin{array}{l} resultIndex \leftarrow \mathbf{ORIGIN} \\ \text{for } rowIndex \in indices \\ \quad \left\| \begin{array}{l} row \leftarrow \text{str2vec}\left(XFMR\_Params_{rowIndex}\right) \\ \text{for } colIndex \in \mathbf{ORIGIN} .. \text{length}(row) - 1 + \mathbf{ORIGIN} \\ \quad \left\| \begin{array}{l} result_{resultIndex} \leftarrow row_{colIndex} \\ resultIndex \leftarrow resultIndex + 1 \end{array} \right. \\ result_{resultIndex} \leftarrow 13 \\ resultIndex \leftarrow resultIndex + 1 \\ result_{resultIndex} \leftarrow 10 \\ resultIndex \leftarrow resultIndex + 1 \end{array} \right. \\ result \end{array} \right.
$$

$\text{WRITEBIN}(\text{“Flyback\_Rev6gP.txt”}, \text{“byte”}, 0, XFMR\_Bin) = 0$