Slide 2 **Overview**

This presentation introduces a circuit simulation model that includes both inductive coupling and mutual resistance effects.

I'll start by reviewing the concepts of mutual inductance and mutual resistance.

Because coupling and loss effects are largely determined by what happens outside of the core, this model works well even with nonlinear cores in cases where saturation is avoided.

The model parameters are extracted in the frequency domain, but the models also work well in the time domain.

The modeling approach is particularly suited for cases not covered by Dowell's method such as certain integrated magnetics and transformers with multiple output windings.

A transformer for a phase-shifted bridge converter is used as an example to illustrate the modeling approach.

Slide 3 **Dowell’s Method Limitations**

Dowell’s Method is used to calculate the ac resistance of transformer windings. These ac resistance values can be used to create winding models that typically include L-R networks such as Foster networks. If, however, Dowell’s Method doesn’t apply to a particular design, then models based on that approach won’t be accurate. Still, as engineers we often have to work with less information than we would like to have, and having a model that predicts some winding losses may be better than not even trying.

Here are some things to consider. Dowell’s Method assumes infinite magnetizing inductance, which means that the primary and secondary windings have equal and opposite amp-turns. It isn’t intended for low-permeability or gapped cores because primary and secondary amp-turns will be unequal. There may also be fringing losses that wouldn’t be accounted for.

Another way of looking at this is that Dowell’s Method assumes that there is only one independent current variable so that all of the currents are scaled by turns ratios.

Interleaved windings are allowed if they are connected in series.

Multiple outputs with independent load currents not allowed.

Windings connected in parallel not allowed because the current sharing ratio is unknown.

The modeling method that I will be discussing today is more general, and doesn’t have the limitations of models that are based on Dowell’s Method.

Slide 4 **Magnetic Coupling Review**

Two windings are coupled when some of the magnetic flux produced by currents flowing in either of the windings passes through both windings.

Only part of the flux produced by a current in one winding reaches other windings.

Flux which doesn’t pass through both windings is called the leakage flux.

Magnetic coupling can also be modeled in terms of self and mutual impedances.

Slide 5 **Self and Mutual Impedance Equations**

These are the basic linear equations for two coupled windings that show how the voltages and currents are related. Mutual inductance is often considered, but if winding losses are to be accurately modeled then we need to consider how losses in one winding are affected by the currents in other windings because of mutual resistance. When both the mutual resistance and the impedance due to mutual inductance are considered together, the resulting complex number can be called mutual impedance. Capacitive effects could also be considered as part of the mutual impedance, but in this presentation, I'm just considering resistance and inductance.

The equation for power loss has term for the power loss due to the currents in each winding, but also has a term for the mutual resistance losses due to the currents in both windings. The asterisk denotes the complex conjugate.

Slide 6 **Impedance Matrix Equation for N Windings**

A set of coupled windings can be modeled with a matrix equation that relates frequency-domain winding voltages and currents with an impedance matrix.

The values of impedance matrix elements can be obtained through FEA simulations or extracted from measurements.

The impedance matrix values vary with frequency.

It should be noted that mutual impedance values are symmetric. In other words, Z_{12} always equals Z_{21}. This can be proven through conservation of energy arguments.

Slide 7 **ANSYS Maxwell Impedance Matrix Results**

This is an example of the impedance matrix data produced by the ANSYS Maxwell finite element magnetics software.

Series Resistance and inductance value are produced at each frequency that is simulated. I created a Mathcad file that parses this this data and then processes it to produce the model coefficients. I won't be going into the details of how the data is processed today, but I have posted some example files on my personal website. There is a link at the end of this presentation. I also gave a presentation on this at the PSMA magnetics workshop last June.

Slide 8 **Transformer Comparison**

The modeling method that I'm discussing can be applied to any winding or any set of coupled windings. For this presentation, I'm analyzing a transformer for a phase-shifted bridge power converter

I used a Maxwell 2D radial model for an ETD49 core, which has a round center leg.

I wound two transformers with different insulation thicknesses between the windings. I started testing with the transformer with the thinner insulation, but this appeared to produce too much EMI that seemed to be interfering with the control circuit, so I wound another transformer with more insulation. It turned out that a mis-wired connection was the real issue, but I completed the testing with the second transformer.

Both transformers have two primary windings, the first on the inner layer and the second on the outer layer. These windings are connected in parallel. This structure, in which the secondary windings are between the primary windings, reduces the leakage inductance and the winding losses.

This model can predict the current sharing between the two primary windings, the couplings among the windings, and the winding losses in both the frequency and time domains.

There are three reasons that this type of transformer isn't suited for Dowell's analysis. First, Dowell assumes an infinitely large magnetizing inductance, which isn't appropriate for a transformer with a gapped core. Second, it has windings that are connected in parallel, and third, it has secondary windings that can be independently loaded.

Slide 10 **Self Inductances**

The calculated self-inductances are relatively constant, but they do decrease somewhat at higher frequencies. The two primary inductances are nearly equal, but the outer primary winding has slightly more inductance. Similarly, the outer secondary winding has a little more inductance than the inner secondary winding. The secondary windings have two layers, and this causes them to have a greater drop in inductance as frequency is increased compared to the primary windings, which only have one layer.

Slide 10 **Self Resistances**

As expected, the self-resistances of the windings increase with frequency. The mid portions of the curves where the slopes are highest are due to proximity effects. At higher frequencies, the skin effect takes over and the slopes are reduced.

As expected, the outer primary winding has more resistance than the inner primary windings. The two secondary windings are in the middle of the winding stack and their resistance values are closer to each other.

Slide 11 **Mutual Inductances**

The mutual inductance between the two primary windings is a little less than the primary inductances, just as the mutual inductance between the two secondary windings is a little less than the secondary inductances.

The mutual inductances between primary and secondary windings are grouped together and they are a little less than the geometric mean of the primary and secondary inductances.

Slide 12 **Mutual Resistances**

The mutual resistance curves are a little more interesting. First, the mutual resistance is zero at dc, so they don't have the low-frequency asymptotes the that the self-resistances do because of the dc resistances. As with the self-resistances, the proximity effect dominates at lower frequencies and the skin effect dominates at higher frequencies.

There are some circuit simulators that model mutual resistance, but the values are not frequency dependent. This is useful if you are modeling a transformer that operates at a fixed frequency with sinusoidal currents.

The mutual resistance between the first and second windings is the highest, and the mutual resistance between the first and fourth windings is the lowest, and that mutual resistance becomes negative at higher frequencies. I'll discuss the significance of that later.

Slide 13 **Definition of Leakage Impedance**

The leakage impedance is the impedance measured at one winding when another winding is shorted.

Leakage impedances are a function of self and mutual impedances as shown in the equation.

Consequently, leakage impedances are a property of a pair of windings and generally can't be split up and assigned to individual windings when there are more than two windings. This is the reason that winding models based on Foster networks are best suited to two-winding transformers. The plots in the following slides show the results of the Maxwell simulations.

Slide 14 **Leakage Inductances**

The leakage inductances are due to the imaginary part of the leakage impedances. As expected, the leakage inductances increase as the spacing between the windings increases.

Also as expected, the leakage inductances decrease with increasing frequency. This effect is caused by the skin and proximity effects that re-position the current flowing in the windings.

Slide 15 **Leakage Resistances**

The leakage resistances are due to the real part of the leakage impedances. Unlike the leakage inductances, the leakage resistances aren't substantially affected by the distances between the windings.

The leakage resistances also depend on any windings that are in between the shorted and measured windings.

The leakage resistances are the main drivers of winding losses. Foster networks can accurately predict the ac losses in two-winding transformers, but they don't work as well when more windings are added.

At low frequencies, the leakage resistance is about equal to the dc resistance of the measured winding plus the dc resistance of the shorted winding reflected through the square of the turns ratio.

At higher frequencies, the situation becomes more complicated.

Slide 6 **Inductive Coupling Coefficients**

Inductive coupling coefficients are familiar to magnetics designers. They can be calculated from self and mutual inductances or self and leakage inductances.

As expected, the coupling coefficients increase as frequency increases. This correlates with the leakage inductances decreasing as frequency increases.

Inductive coupling coefficients are negative when the mutual inductance is negative. The coupling polarity can be reversed by reversing the reference direction of one of the currents.

The inequality ensures that the total stored energy is always positive for two windings.

A more restrictive criterion is needed to ensure that a coupled inductance model remains passive for more than two windings as is explained in reference 9. As set of coupled windings can mathematically behave as an infinite energy source if certain physically impossible sets of coupling coefficients are used. The overall stability criterion is that the inductance matrix that results from the coupling coefficients is positive definite. Similarly, the coupling coefficient matrix must be positive definite. Ultimately what needs to be prevented are system equation terms in the time domain where e is raised to a positive power with respect to time.

Slide 17 **Resistive Coupling Coefficients**

Just as with mutual inductances, a coupling coefficient for mutual resistance k^R can be defined as explained in reference 10. That paper is discussing power dissipation in inductive heating, but it turns out that the concept also applies to transformers.

The form of the equation for computing resistive coupling coefficients is the same as for inductive coupling coefficients.

The resistive coupling coefficient is negative when the mutual resistance is negative. The sign of the resistive coupling coefficient can be reversed by changing the reference direction of one of the currents.

There are transformers where the resistive coupling coefficient changes sign as a function of frequency, but this transformer doesn't show that behavior.

Resistive coupling coefficients head to zero as the frequency decreases.

The inequality ensures that the total dissipated power is always positive for two windings.

In this model, the resistive coupling coefficients aren't specified, but are just analyzed. Since the inductive coupling is kept passive through the constraints described in reference 9, the resistive coupling is just a consequence of the inductive coupling and will always be passive.

Slide 18 **Effects of Mutual Resistance on Leakage Resistance**

The leakage resistance (green) is less than the sum of the self-resistance R22 and the reflected self-resistance R33 (pink).

There is a significant reduction of the leakage resistance due to the mutual resistance between these adjacent windings. The reduction in leakage resistance increases as the mutual resistance coupling increases.

This is very much like what happens with leakage inductance. The measured inductance of a winding is decreased when another winding is shorted. Similarly, for most situations at low and medium frequencies, the ac resistance measured at a winding is decreased when another winding is shorted. For some situations, the high frequency resistance measured at a winding will actually increase when another winding is shorted. As shown in later slides, that occurs when the mutual resistance coupling is negative.

Slide 19 **Effects of Mutual Resistance on Leakage Resistance**

This transformer has more insulation between the layers, but the resistance reduction due to mutual resistance is even stronger. I'm working on understanding the various mutual resistance effects and how they vary with different winding constructions.

Slide 20 **Effects of Mutual Resistance on Leakage Resistance**

The leakage resistance (brown) is less than the sum of the first primary self-resistance R11 and the second primary self-resistance R44 (blue) reflected through the 1:1 turns ratio.

These coils are not adjacent, and the leakage resistance coupling actually becomes negative. This means that the losses are higher than the sum of the two winding resistances. Note the order of the curves flips at the frequency when the resistive coupling coefficient changes sign.

Slide 21 **Effects of Mutual Resistance on Leakage Resistance**

This transformer doesn’t have the same level of negative resistance coupling and the so the increase in resistance at higher frequencies isn’t as pronounced as what is shown in the previous slide.

Slide 22 **Impedance of an Isolated Winding**

I decided to compare the impedance of an isolated winding as I was winding the second transformer to what the impedance was after the transformer was completed to see what I might learn. I performed a Maxwell simulation of that winding and also simulated the completed transformer. I modified my Mathcad file to handle just one winding.

Slide 23 **Impedance of an Isolated Winding**

The ac resistance increases as expected, and the inductance is essentially constant.

Slide 24 **Impedance of Same Winding Within a Transformer**

Here are the details of the completed transformer. As is typical, the winding build-up is a little greater than the sum of the copper and insulation layers. I increased the spacing caused by 2-mil thick Nomex to 3 mils based on measurements of the winding build that I made as I wound the transformer. I used 20 AWG wire soldered to the ends of the foil windings to connect to the bobbin terminals. The wire was flattened to reduce the lumping of the windings. I used two-inch pieces of litz wire to connect the bobbin terminals to the HP4194A network analyzer during impedance measurements. I found that when flying lead connections were required for transformers using foil windings that using flat litz wire soldered to the foil windings reduced power losses by several watts compared to using stranded lead wires.

Slide 25 **Impedance Comparison**

The inductance of the first winding stayed the same when I added the additional windings, but the ac resistance increased significantly. Note that when the additional windings are added, the effective resistance of the original winding is increased, even though there are no currents flowing in the other windings. This is because the magnetic field created by the original winding creates eddy current losses in the other windings.

Slide 26 **Impedance of an Isolated Winding**

Here is an equivalent circuit model of the isolated winding. The conventional way to model this is with a series arrangement of several inductors with a resistor connected in parallel with each inductor. This is called a Foster type 1 network. In this case, I used a different approach. I still have multiple inductor-resistor pairs, but these pairs are coupled to a single inductor instead of being connected in series. These extra inductors are called auxiliary inductors, and they all have the same inductance as the inductor that is wound on the bobbin. The values of the couplings and the parallel resistors are adjusted to shape the impedance. This alternative equivalent circuit allows mutual resistances between windings to be accurately modeled by coupling the auxiliary inductors to each of the inductors that represent the physical windings.

Slide 27 **Equivalent Circuit Model of Winding Impedance**

This slide shows the that the isolated winding can be modeled with the equivalent circuit. I also show the results of modeling that same winding using an equivalent circuit for the whole transformer.

Slide 28 **An Equivalent RL Circuit for a Four-Winding Transformer**

The circuit model is based on methods described in references 11 and 12.

The physical windings are represented by L1, L2, L3 and L4.

Each physical winding is accompanied by a set of auxiliary windings. Each auxiliary winding is shunted by a resistor. The resistors model the ac losses, and they also reduce the inductances as the frequency is increased.

Increasing the number of auxiliary windings increases the frequency range of the model. Having three auxiliary windings is generally adequate to obtain accurate results up to 10 MHz.

Each physical winding is coupled to each of the auxiliary windings, but the auxiliary windings are not coupled to each other.

Some of the couplings could be negative.

The auxiliary windings are assigned to have the same inductance as their associated physical winding because this makes the coefficient extraction process easier.

The parameter values were determined by a solver in Mathcad that attempts to match the performance of the model to impedance matrix data imported from Maxwell finite element simulations.

Slide 29 **Transformer Equivalent Circuit Model**

This shows how the transformer model is implemented in LTspice. My Mathcad file exports a text file that is added with a .inc statement to avoid cluttering the schematic. I added the contents of the text file to the slide.

The same type of model could be implemented in circuit simulators that use mutual inductances instead of coupling coefficients.

Slide 30 **Model Coupling Coefficients**

The couplings among the physical windings are high, but the couplings among the physical windings and the auxiliary inductances are low.

A matrix of all the coupling coefficients that are used in the model can be used to check the stability of the model.

All the eigenvalues of the coupling matrix must non-negative to ensure stability of the model.

The solver checks for stability and rejects unstable solutions.

The model is reduced-order because not all the couplings are included. The model in reference 11 includes all of the couplings, but I haven't found that to be necessary. A description of the reduced-order model is given in reference 12, which is a discussion of the paper in reference 11 that I submitted. I had some questions and comments for the authors. The paper and the discussion were published in 2000. I have found the model useful in my work in industry, and now that I am at Utah State University, I have been refining the modeling approach.

Slide 31 **Self Resistance and Inductance Capacitive Correction**

This modeling approach can be based on measurements as well as on finite element simulations, but I found that winding capacitances limited the frequency range where I could get accurate inductance measurements. In order to get better measurements, I developed a method of cancelling most of the effects of the first parallel resonance due to winding capacitance. It works well for inductors and typical transformers if the second resonance is significantly higher than the first resonance.

I typically gather series resistance and inductance data with an LCR meter or a network analyzer.

I start the correction by computing the parallel capacitance from the inductance and the first resonant frequency. The formula for each step after that is listed in the slide.

I convert the series data to parallel resistance and reactance data at each frequency. I also compute the reactance of the parallel capacitance at each frequency. The reactance of the parallel inductance is calculated by removing the effect of the parallel capacitive reactance.

The parallel resistance and corrected parallel inductance are then converter back to the series forms.

The inductance correction is very good. The resistance still shows peaking, but at a higher frequency.

The capacitive cancellation method works better for some windings than others.

Winding capacitances can be added back into the model to reproduce the original measured impedances.

Slide 32 **Self Resistances and Inductances**

This slide and the following five slides show plots of the finite element data, the circuit model results, and measured data from a transformer that I wound. The circuit model almost perfectly matches the finite element data, and the measurements are fairly close to the predicted values.

Slide 33 **Measured Self Resistances and Inductances**

The effects of the winding capacitance aren't totally removed, so I only used measured data up to 600 kHz, compared to 10 MHz for the FEA-based model.

Slide 34 **FEA and Equivalent Circuit Leakage Inductances**

The circuit model tracks the Maxwell leakage inductance data very well. I also added a plot for the Q of the leakage inductance. The Q is defined as the imaginary part of the leakage impedance divided by the real part. Transformers that are designed to have large leakage inductances for resonant converters can have Q values of 200 or more at the operating frequencies, but closely couple transformers like this one commonly have Q values ranging from around 2 to 20 over the frequency range of 10 kHz to 10 MHz.

Slide 35 **Measured and Equivalent Circuit Leakage Inductances**

The equivalent circuit based on the measured leakage impedances tracks the measured data fairly well, but not as good as the Maxwell data.

Slide 36 **FEA and Equivalent Circuit Leakage Resistances**

The equivalent circuit tracks the FEA leakage inductance data very well. The winding losses are primarily caused by the leakage resistances, so the model should be good at predicting the winding losses in transformers using circuit simulators. The time-domain results will be accurate when the frequency-domain results are accurate.

Slide 37 **Measured and Equivalent Circuit Leakage Resistances**

The equivalent circuit based on the measured leakage impedances tracks the measured data fairly well, but not as good as the Maxwell data. In this case, the circuit model tends to over-predict the losses at high frequencies. The circuit model parameter extraction can be adjusted to match different frequency ranges more closely by adjusting the weighting parameters in the Mathcad solver.

Slide 38 **FEA and Equivalent Circuit Mutual Resistances**

The equivalent circuit tracks the FEA mutual resistance data very well.

Slide 39 **Measured and Equivalent Circuit Mutual Resistances**

The equivalent circuit matches the mutual resistances based on measured data best at the middle frequencies. The mutual impedance data here is calculated from the self and leakage impedance measurements because it can’t be measured directly.

Slide 40 **FEA and Equivalent Circuit Mutual Resistance Coupling**

Since the equivalent circuit mutual resistance results closely match the FEA data, the mutual resistance coupling results will also be very good.

Slide 41 **Measured and Equivalent Circuit Mutual Resistance Coupling**

The mutual resistance plots based on measured data has a more complicated shape than the mutual resistance plots based on FEA data, but the equivalent circuit model still closely tracks the data over most of the frequency range.

Slide 42 **Self-Impedance SPICE Simulation**

This is an LTspice simulation of the self-impedance of the first winding. This is accomplished by injecting a 1A current into the winding and measuring the resulting voltage. The model parameters are set in the .inc file.

Slide 43 **Self-Impedance SPICE Simulation**

An LTspice circuit was created based on the Mathcad parameter extraction. The model parameters are shown on the next slide.

This slide shows that that the self-impedance can be measured by injecting a one-amp current into one of the windings and then measuring the voltage.

The winding resistance is obtained by plotting the real part of the voltage. The inductance is obtained by plotting the imaginary part of the voltage and dividing by the radian frequency.

The SPICE simulations are able to accurately reproduce the results produced in the Mathcad calculations.

Slide 44 **Leakage-Impedance SPICE Simulation**

In this simulation, the leakage impedance is obtained by injecting 1A of current into the first winding and measuring the voltage while the fourth winding is shorted.

Slide 45 **Leakage-Impedance SPICE Simulation**

I performed simulations for FEA-based equivalent circuits for the two transformers that I built. The leakage resistance is the same for both transformers. As expected, the leakage inductance for the transformer with the thickest insulation has the highest leakage inductance.

The leakage inductance for both transformers decreases with frequency, but the curves have nearly the same shape. This is because the energy stored in the windings is similar, but the energy stored in the volume of the thicker insulation is greater than for the thinner insulation. This also increases the Q of the leakage inductance in the transformer with the thicker insulation.

Slide 46 **Phase-Shifted Bridge Converter**

I took a TI phase-shifted bridge demo board, removed the transformer and ZVS inductor and added my own rectifier assembly. I also wound two additional ZVS inductors to use in my testing. The demo board is intended to operate from a 400V supply, but I dropped that to 150V so that it would be compatible with the foil-wound transformers that I wanted to test. It isn’t practical to get as many turns on foil-wound transformers as can be done with wire-wound transformers.

Slide 47 **Phase-Shifted Bridge Converter Power Stage**

This is the schematic diagram of the power stage of the phase-shifted bridge converter as implemented in an LTspice simulation. It shows the two primary windings connected in parallel. Having the inner and outer windings connected in parallel reduces the leakage inductances, but the current sharing will be unequal. The current sharing shown in simulations is compared to measurements in later slides.

I used diode clamps instead of snubbers on the output rectifiers.

Slide 48 **Simulation and Measured Test Results, Lzvs = 2.4 µH**

SPICE based on Measured Data

I performed simulations using two different ZVS inductors with SPICE models based on FEA simulations and on measured data. Four different loading conditions were used. Both types of SPICE models were able to model cross regulation and accurately track the dc values of the output voltages.

One particular load condition produced very different results than the others. The voltage feedback was connected to the positive output. When the positive output was lightly loaded and the negative output was fully loaded, the positive output of the converter was able to stay in regulation with a narrow duty cycle, but the negative output voltage collapsed from 30 volts to 2 volts.

Slide 49 **Simulation and Measured Test Results, Lzvs = 2.4 µH**

SPICE based on Measured Data

The SPICE simulation based on measured LCR data also gave accurate results of the DC output voltages.

Slide 50 **Simulation and Measured Test Results, Lzvs = 10.7 µH**

SPICE based on FEA Data

Slide 51 **Simulation and Measured Test Results, Lzvs = 10.7 µH**

SPICE based on Measured Data

Again, the SPICE simulation based on measured LCR data produced results that are very similar to the FEA-based results.

Slide 52 **Transformer Primary Currents Lzvs = 2.4 µH, FEA-Based SPICE**

The primary currents were measured with a Tektronix P6021 current probe and a Tektronix 134 amplifier. The rated bandwidth for the combination is 35 MHz.

The inner primary winding carried 24% more current than the outer primary winding in the oscilloscope measurements but the SPICE simulation only predicted a 15% increase for the inner primary winding. It is possible that the circuit board traces contributed to some of the error.

The waveshapes are somewhat different, but the errors in the rms values were less than 10%.

Slide 53 **Transformer Primary Currents Lzvs = 2.4 µH, Meas-Based SPICE**

The rms current errors for the LCR measurement-based simulations were also less than 10%. There is less ringing in the measurement-based simulations compared to the FEA-based simulations, and the waveshapes are actually closer to the oscilloscope waveforms.

The LCR measurement-based simulations predicted worse current sharing than the oscilloscope measurements, while the FEA-based simulations predicted better current sharing than the oscilloscope measurements.

Slide 54 **Transformer Primary Currents Lzvs = 10.7 µH, FEA-Based SPICE**

These waveforms are similar to the FEA-based waveforms for the 2.4 µH ZVS inductor, and the accuracies are also similar. The di/dt current slopes are observably less than with the 2.4 µH ZVS inductor.

Slide 55 **Transformer Primary Currents Lzvs = 10.7 µH, Meas-Based SPICE**

These waveforms are similar to the LCR measurement-based waveforms for the 2.4 µH ZVS inductor, and the accuracies are also similar. Again, the di/dt current slopes are observably less than with the 2.4 µH ZVS inductor.

Slide 56 **Phase-Shifted Bridge Transformer Loss Waveforms, FEA**

Simulations using the FEA-based simulation model were performed for the 2.4 µH ZVS inductor and the 10.7 µH ZVS inductor. The current slopes are clearly reduced by the larger ZVS inductor.

I added up all of the losses in the resistors that represent the dc resistances. These losses are primarily the low-frequency winding losses.

I also added up the losses for the auxiliary resistors, which represent the high-frequency losses. The average power loss values are listed to the left of the plots. Increasing the ZVS inductance has little effect on the low-frequency losses, but cuts the high-frequency losses almost in half.

Slide 57 **Phase-Shifted Bridge Transformer Loss Waveforms, Meas**

The simulations based on the measured LCR data has greater high-frequency losses than the simulations based on the FEA data. This is why the current waveforms have less ringing, and are closer to the oscilloscope waveforms. I presume that the core losses are part of the reason for the increased high-frequency losses. I plan to investigate that further. As with the simulations based on FEA data, these simulations show little difference in the low-frequency losses when the ZVS inductance is increased, but the high frequency losses are also reduced by nearly 50%.

Slide 58, **Maxwell 2D Pulse Test Transformer Model**

It has been recognized that leakage inductance decreases with increasing frequency. It seemed to me that a corollary to that would be that the leakage inductance would appear small at the beginning of a voltage pulse across a winding, and would apparently increase with time. Last year I wound transformer to demonstrate this effect. I didn't have time to repeat the measurements for the foil-wound transformers used in this presentation, but the principle is the same.

Slide 59 **Leakage Inductance Variation with Pulse Width**

A voltage pulse is applied to winding 1 while winding 2 is shorted in order to show how the apparent leakage inductance varies with time.

Since V= L di/dt, we can determine the apparent inductance by dividing the voltage by the time derivative of the current.

$$V = L\frac{di}{dt} \qquad L = \frac{V}{\frac{di}{dt}}$$

Slide 60 **Leakage Inductance Variation with Pulse Width**

As I expected, I found that the slope of the primary current decreased during the time of the pulse.

The apparent leakage inductance can be measured as the voltage across the winding divided by the derivative of the current.

The apparent leakage inductance can be calculated by dividing the instantaneous primary voltage by the derivative of the total primary current. Although it is a little hard to see, the slope of the inductor current drops as the pulse progresses. However, it is much easier to see that the apparent leakage inductance indicated by the red waveform increases from 4 µH to over 10 µH in 1.6 microseconds.

The range for the effective leakage inductance in the time domain is close to the range in the frequency domain.

A practical application of this result is that the di/dt seen during rectifier diode reverse recovery is due to the transformer leakage inductance at high frequencies, not at lower frequencies where it might more commonly be measured.

Slide 61 **Diode Reverse Recovery Test Circuit**

This circuit can be used to simulate the reverse-recovery of diodes. A diode is modeled using the modified charge control model which is better at modeling reverse recovery than the model included with standard SPICE.

The diode reverse recovery was simulated with the full mutual resistance equivalent circuit model and then with the mutual resistance turned off.

Slide 62 **Diode Reverse Recovery Comparison**

The mutual resistance equivalent circuit model reduces the effective inductance for short pulses and increases the peak reverse recovery current compared to a model with no mutual resistances. My recommendation is that if you are modeling reverse recovery of diodes driven by a transformer, and the transformer model has fixed leakage inductances, the results will be more accurate if leakage inductances measured at high frequencies such as 1-10 MHz are used instead of using values obtained at more commonly used frequencies such as 10-100 kHz.

Slide 63 **Conclusions**

This mutual impedance circuit model can be made to match FEA results very closely.

The mutual impedance circuit model also matches measured data fairly well, but the range of frequencies where accurate measurement results can be obtained are more limited than FEA simulations because of winding capacitances.

Compensating for the winding capacitances can extend the frequency range of accurate measurements.

The SPICE circuit models produced from FEA simulations are more accurate at predicting the current sharing than the models produced by LCR measurements.

The SPICE circuit models produced from both the FEA simulations and LCR measurements accurately predict the dc measurements.

All of the model variations were able to capture a loading condition of concern.

Slide 64 **Conclusions (continued)**

The leakage inductance for closely-coupled winding pairs decreases with frequency.

The inductance decrease is due to skin and proximity effects.

The effective leakage inductance for pulsed waveforms can be determined by dividing the applied voltage by the time derivative of the current.

The effective leakage inductance for short pulses is less than for longer pulses.

The currents produced by the reverse recovery of fast diodes connected to transformer outputs depend on the high-frequency leakage inductance values.

Example calculation and simulation files can be found on my personal website at:

http://www.verimod.com/resources.html