Four-Winding Transformer Model Coefficient Extraction from Measured Data

Bryce Hesterman
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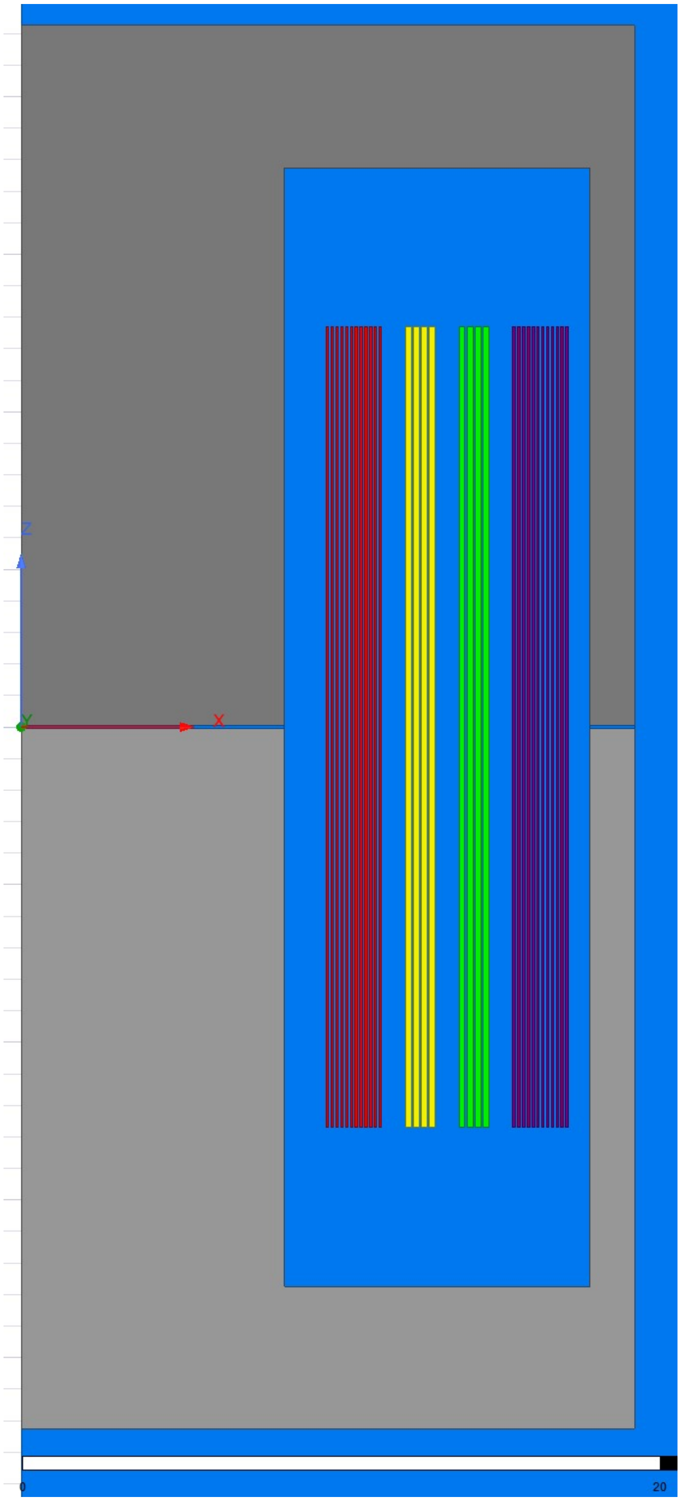


Winding 1: 12 T 0.003" by 1" copper foil

Winding 2: 4 T 0.007" by 1" copper foil

Winding 3: 4 T 0.007" by 1" copper foil

Winding 4: 12 T 0.003" by 1" copper foil

2mil Nomex between each layer.

10 layers 2 mil Nomex between windings.
(Nomex modeled as 3 mil based on measurements of the wound bobbin)

Core: ETD49-25-16 3C97
Gap: 3 mil

Bobbin: TDK B66368B1020T001

Fig. 1. Transformer construction details.

Unit definitions. $m\Omega := \dfrac{\Omega}{1000}$ $nH := H \cdot 10^{-9}$

Measured dc resistances. $Rdc := \begin{bmatrix} 9.015 \\ 2.715 \\ 3.326 \\ 13.44 \end{bmatrix} \cdot m\Omega$

Winding	DC Resistance (mohm)	Magnetizing Inductance (uH)
Winding A	9.0849	192.9
Winding B	1.55713	21.4453
Winding C	1.76643	21.4486
Winding D	14.18	192.965

Low-frequency resistances and inductances.

$$R0 := \begin{bmatrix} 0.041787199 \\ 0.006944125 \\ 0.007056362 \\ 0.046712414 \end{bmatrix} \cdot \Omega \qquad L0 := \begin{bmatrix} 0.000191621 \\ 0.0000213764 \\ 0.0000214084 \\ 0.000192131 \end{bmatrix} \cdot H \qquad f0 := 10715.193 \cdot kHz \quad \omega 0 := 2 \cdot \pi \cdot f0$$

$$Rb := \begin{bmatrix} Rdc_1 & 0 & 0 & 0 \\ 0 & Rdc_2 & 0 & 0 \\ 0 & 0 & Rdc_3 & 0 \\ 0 & 0 & 0 & Rdc_4 \end{bmatrix}$$

Low-frequency leakage inductances.

$Lleak12\_0 := 0.00000160316 \cdot H$ $Lleak13\_0 := 0.00000262227 \cdot H$ $Lleak14\_0 := 0.00000293719 \cdot H$

$Lleak23\_0 := 0.00000022953 \cdot H$ $Lleak24\_0 := 0.000000323769 \cdot H$ $Lleak34\_0 := 0.000000230253 \cdot H$

$$Kb := \begin{bmatrix} 1 & \sqrt{1 - \dfrac{Lleak12\_0}{L0_1}} & \sqrt{1 - \dfrac{Lleak13\_0}{L0_1}} & \sqrt{1 - \dfrac{Lleak14\_0}{L0_1}} \\ \sqrt{1 - \dfrac{Lleak12\_0}{L0_1}} & 1 & \sqrt{1 - \dfrac{Lleak23\_0}{L0_2}} & \sqrt{1 - \dfrac{Lleak24\_0}{L0_2}} \\ \sqrt{1 - \dfrac{Lleak13\_0}{L0_1}} & \sqrt{1 - \dfrac{Lleak23\_0}{L0_2}} & 1 & \sqrt{1 - \dfrac{Lleak34\_0}{L0_3}} \\ \sqrt{1 - \dfrac{Lleak14\_0}{L0_1}} & \sqrt{1 - \dfrac{Lleak24\_0}{L0_2}} & \sqrt{1 - \dfrac{Lleak34\_0}{L0_3}} & 1 \end{bmatrix}$$

$$Kb = \begin{bmatrix} 1.0000 & 0.9958 & 0.9931 & 0.9923 \\ 0.9958 & 1.0000 & 0.9946 & 0.9924 \\ 0.9931 & 0.9946 & 1.0000 & 0.9946 \\ 0.9923 & 0.9924 & 0.9946 & 1.0000 \end{bmatrix}$$

All of the eigenvalues must be positive to have a stable model [2]. $\mathrm{eigenvals}(Kb) = \begin{bmatrix} 3.981436 \\ 0.009303 \\ 0.005569 \\ 0.003692 \end{bmatrix}$

$$Lb := \begin{bmatrix} L0_1 & Kb_{1,2}\cdot\sqrt{L0_1\cdot L0_2} & Kb_{1,3}\cdot\sqrt{L0_1\cdot L0_3} & Kb_{1,4}\cdot\sqrt{L0_1\cdot L0_4} \\ Kb_{1,2}\cdot\sqrt{L0_1\cdot L0_2} & L0_2 & Kb_{2,3}\cdot\sqrt{L0_2\cdot L0_3} & Kb_{2,4}\cdot\sqrt{L0_2\cdot L0_4} \\ Kb_{1,3}\cdot\sqrt{L0_1\cdot L0_3} & Kb_{2,3}\cdot\sqrt{L0_2\cdot L0_3} & L0_3 & Kb_{3,4}\cdot\sqrt{L0_3\cdot L0_4} \\ Kb_{1,4}\cdot\sqrt{L0_1\cdot L0_4} & Kb_{2,4}\cdot\sqrt{L0_2\cdot L0_4} & Kb_{3,4}\cdot\sqrt{L0_3\cdot L0_4} & L0_4 \end{bmatrix}$$

$$Lb = \begin{bmatrix} 0.000192 & 0.000064 & 0.000064 & 0.00019 \\ 0.000064 & 0.000021 & 0.000021 & 0.000064 \\ 0.000064 & 0.000021 & 0.000021 & 0.000064 \\ 0.00019 & 0.000064 & 0.000064 & 0.000192 \end{bmatrix} \boldsymbol{H}$$

All of the eigenvalues must be positive to have a stable model [2]. $\text{eigenvals}(Lb) = \begin{bmatrix} 0.000425 \\ 0.000001 \\ 2.096815\cdot 10^{-7} \\ 9.625842\cdot 10^{-8} \end{bmatrix} \boldsymbol{H}$

Measured self resistances and inductances.

freq (Hz)	R11 (ohm)	L11 (H)	R44 (ohm)	L44 (H)
10715.193	0.04178719867	0.0001916205289	0.04671241422	0.000192131409
18302.061	0.06272154361	0.0001912726653	0.0692200725	0.0001918013037
27701.29	0.06222569317	0.0001911554436	0.07209343422	0.0001916836159
39810.717	0.09198480999	0.0001911050148	0.102979819	0.0001916243321
56234.133	0.1278816588	0.0001910081267	0.1405959068	0.0001915297479
76736.149	0.1797712469	0.000190919197	0.1949086004	0.0001914497732
102920.053	0.2770008175	0.0001908307767	0.2855644492	0.0001913660387
133352.143	0.4007511742	0.0001907549112	0.4226153134	0.0001913025329
178854.553	0.6701177757	0.0001906659949	0.6974338662	0.0001912200841
244061.907	1.19563133	0.0001905219084	1.234659599	0.0001910876188
316227.766	2.056276188	0.0001903619485	2.105731312	0.0001909293793
446683.592	4.868029296	0.0001902735535	4.920953486	0.0001908433143
599100.939	12.42055629	0.0001900955324	12.37329575	0.0001906840366

R22 (ohm)	L22 (H)	R33 (ohm)	L33 (H)
0.006944124521	0.00002137635	0.007056362095	0.00002140835
0.007986122258	0.00002134319	0.008140550216	0.00002137260
0.007744413913	0.00002134890	0.008007481219	0.00002137503
0.01118774712	0.00002134827	0.01129769472	0.00002137233
0.0159683279	0.00002134064	0.01611051273	0.00002136318
0.0234494476	0.00002133584	0.02332700706	0.00002135998
0.03265620712	0.00002133195	0.03270819307	0.00002135129
0.04728250389	0.00002132438	0.04717427632	0.00002134178
0.07791400574	0.00002131240	0.07730891763	0.00002132845
0.1388829139	0.00002129181	0.1372277904	0.00002130604
0.2384191629	0.00002126680	0.234651676	0.00002127927
0.5636460815	0.00002122196	0.5502114316	0.00002123033
1.418133524	0.00002118066	1.368757664	0.00002118953

$f := 1..\mathrm{rows}(freq)$ $\qquad\qquad$ $\omega_f := 2 \cdot \pi \cdot freq_f$

Compute the self impedances from the measured resistance and inductance values.

$$Z11_f := R11_f + 1\mathrm{j} \cdot \omega_f \cdot L11_f \qquad\qquad Z22_f := R22_f + 1\mathrm{j} \cdot \omega_f \cdot L22_f$$

$$Z33_f := R33_f + 1\mathrm{j} \cdot \omega_f \cdot L33_f \qquad\qquad Z44_f := R44_f + 1\mathrm{j} \cdot \omega_f \cdot L44_f$$

Measured Leakage Impedance Data

Rleak12 (ohm)	Lleak12 (H)
0.04512942785	0.000001603158943
0.04673578372	0.00000159068656
0.04907458177	0.000001584461899
0.05250796038	0.000001576683193
0.05759651968	0.000001567886606
0.06516030428	0.000001559028312
0.07403153904	0.000001550450017
0.08440558017	0.00000154309688
0.1022114152	0.000001534314055
0.1304070957	0.000001524380546
0.166748374	0.000001514995684
0.2376279251	0.000001500433018
0.3222962254	0.00000148559357

Rleak13 (ohm)	Lleak13 (H)
0.0490694718	0.000002620006842
0.05172300873	0.000002600220459
0.05552504152	0.000002590047984
0.06140132061	0.000002578474655
0.07035614175	0.000002565790136
0.08394394032	0.000002552604147
0.1022750931	0.000002540354264
0.1254060067	0.000002528767813
0.1674550014	0.000002513465178
0.2384710676	0.000002493546854
0.3320798797	0.000002471995642
0.5164686328	0.000002434025308
0.7437961411	0.000002390947904

Rleak14 (ohm)	Lleak14 (H)
0.02853541651	0.000002934095797
0.03175883132	0.0000290793781
0.03614130272	0.000002894792219
0.04282300186	0.000002880713085
0.05304812272	0.000002866917434
0.06892993898	0.000002853672269
0.09090173773	0.000002840849626
0.120019542	0.000002829013402
0.1745095553	0.000002812554034
0.2695306177	0.00000278943383
0.3967233359	0.000002762768715
0.6538814088	0.000002713482538
0.9753027164	0.000002655637915

Rleak23 (ohm)	Lleak23 (H)
0.007512691223	0.0000002297458
0.007729034299	0.0000002270733
0.008046146728	0.0000002278112
0.008520889218	0.0000002272482
0.009380434835	0.0000002259805
0.01079084786	0.0000002245711
0.01228145007	0.0000002230632
0.01405224695	0.0000002218058
0.01726766509	0.0000002202888
0.02235001587	0.0000002184788
0.02906664429	0.0000002166434
0.04199087913	0.0000002136568
0.05642637102	0.0000002105126

Rleak24 (ohm)	Lleak24 (H)
0.005299220687	0.0000003230977
0.005536091852	0.0000003204122
0.006087269218	0.0000003198572
0.00665876063	0.0000003187602
0.007714483587	0.0000003170355
0.009555749177	0.0000003154062
0.0116361673	0.0000003138264
0.01426664432	0.0000003124210
0.01918216687	0.0000003106321
0.02754851631	0.0000003083143
0.03879454425	0.0000003057623
0.06139807826	0.0000003012767
0.08814190948	0.0000002963243

Rleak34 (ohm)	Lleak34 (H)
0.005645671754	0.0000002301729
0.00576259567	0.0000002280485
0.00609234097	0.0000002284519
0.006431741694	0.0000002277328
0.007038111208	0.0000002266335
0.008147242483	0.0000002253348
0.009274061298	0.0000002244383
0.01038292169	0.0000002232206
0.01266842374	0.0000002221085
0.01617353412	0.0000002208489
0.0209289736	0.0000002196109
0.03018010268	0.0000002177359
0.04087401373	0.0000002158540

Compute the leakage impedances from the measured leakage resistance and inductance values.

$$Zleak12_f := Rleak12_f + 1j \cdot \omega_f \cdot Lleak12_f \qquad Zleak13_f := Rleak13_f + 1j \cdot \omega_f \cdot Lleak13_f$$

$$Zleak14_f := Rleak14_f + 1j \cdot \omega_f \cdot Lleak14_f \qquad Zleak23_f := Rleak23_f + 1j \cdot \omega_f \cdot Lleak23_f$$

$$Zleak24_f := Rleak24_f + 1j \cdot \omega_f \cdot Lleak24_f \qquad Zleak34_f := Rleak34_f + 1j \cdot \omega_f \cdot Lleak34_f$$

Compute the mutual impedances.

$$Z12_f := \sqrt{\left(Z11_f - Zleak12_f\right) \cdot Z22_f} \qquad Z13_f := \sqrt{\left(Z11_f - Zleak13_f\right) \cdot Z33_f}$$

$$Z14_f := \sqrt{\left(Z11_f - Zleak14_f\right) \cdot Z44_f} \qquad Z23_f := \sqrt{\left(Z22_f - Zleak23_f\right) \cdot Z33_f}$$

$$Z24_f := \sqrt{\left(Z22_f - Zleak24_f\right) \cdot Z44_f} \qquad Z34_f := \sqrt{\left(Z33_f - Zleak34_f\right) \cdot Z44_f}$$

Compute the mutual resistances.

$$R12_f := \mathrm{Re}\left(Z12_f\right) \qquad R13_f := \mathrm{Re}\left(Z13_f\right) \qquad R14_f := \mathrm{Re}\left(Z14_f\right)$$

$$R23_f := \mathrm{Re}\left(Z23_f\right) \qquad R24_f := \mathrm{Re}\left(Z24_f\right) \qquad R34_f := \mathrm{Re}\left(Z34_f\right)$$

Compute the mutual inductances.

$$L12_f := \frac{\mathrm{Im}\left(Z12_f\right)}{\omega_f} \qquad L13_f := \frac{\mathrm{Im}\left(Z13_f\right)}{\omega_f} \qquad L14_f := \frac{\mathrm{Im}\left(Z14_f\right)}{\omega_f}$$

$$L23_f := \frac{\mathrm{Im}\left(Z23_f\right)}{\omega_f} \qquad L24_f := \frac{\mathrm{Im}\left(Z24_f\right)}{\omega_f} \qquad L34_f := \frac{\mathrm{Im}\left(Z34_f\right)}{\omega_f}$$

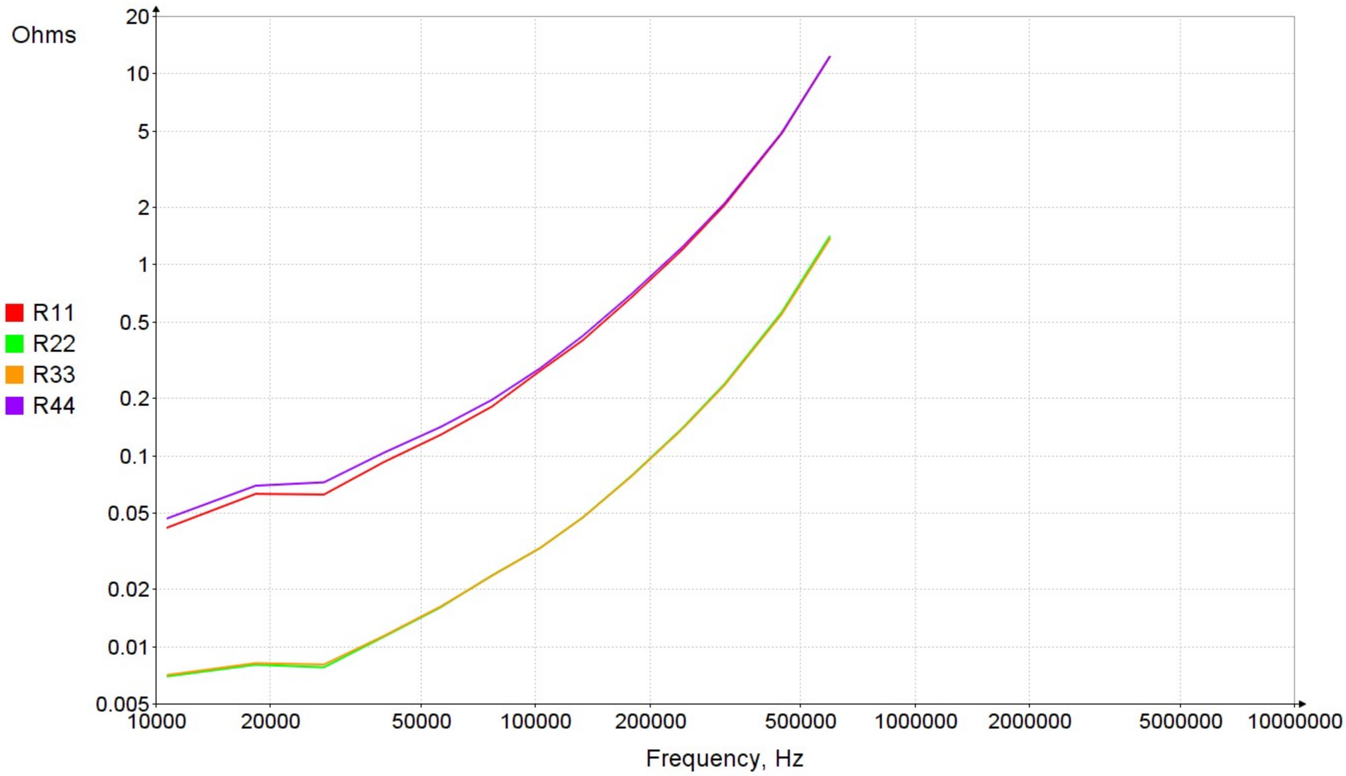


Fig. 2. Measured self resistances.

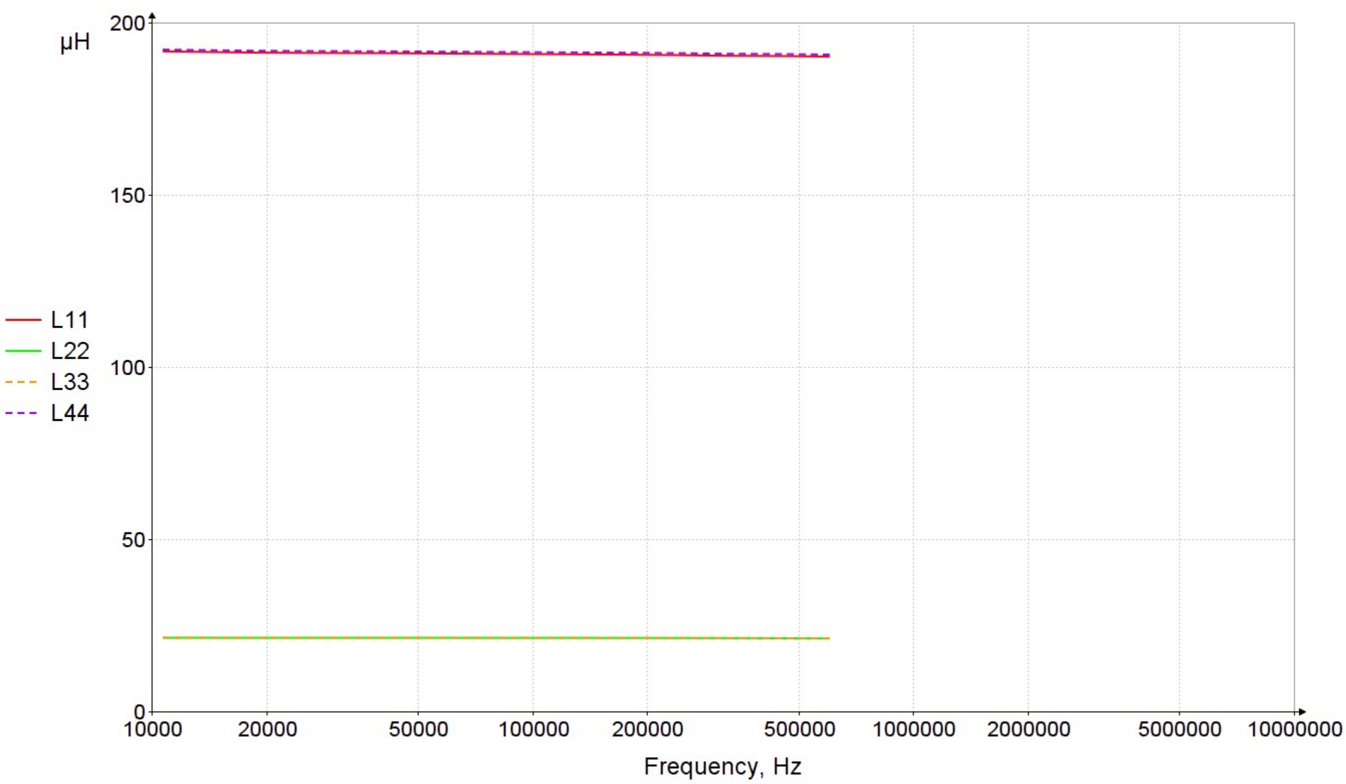


Fig. 3. Measured self inductances.

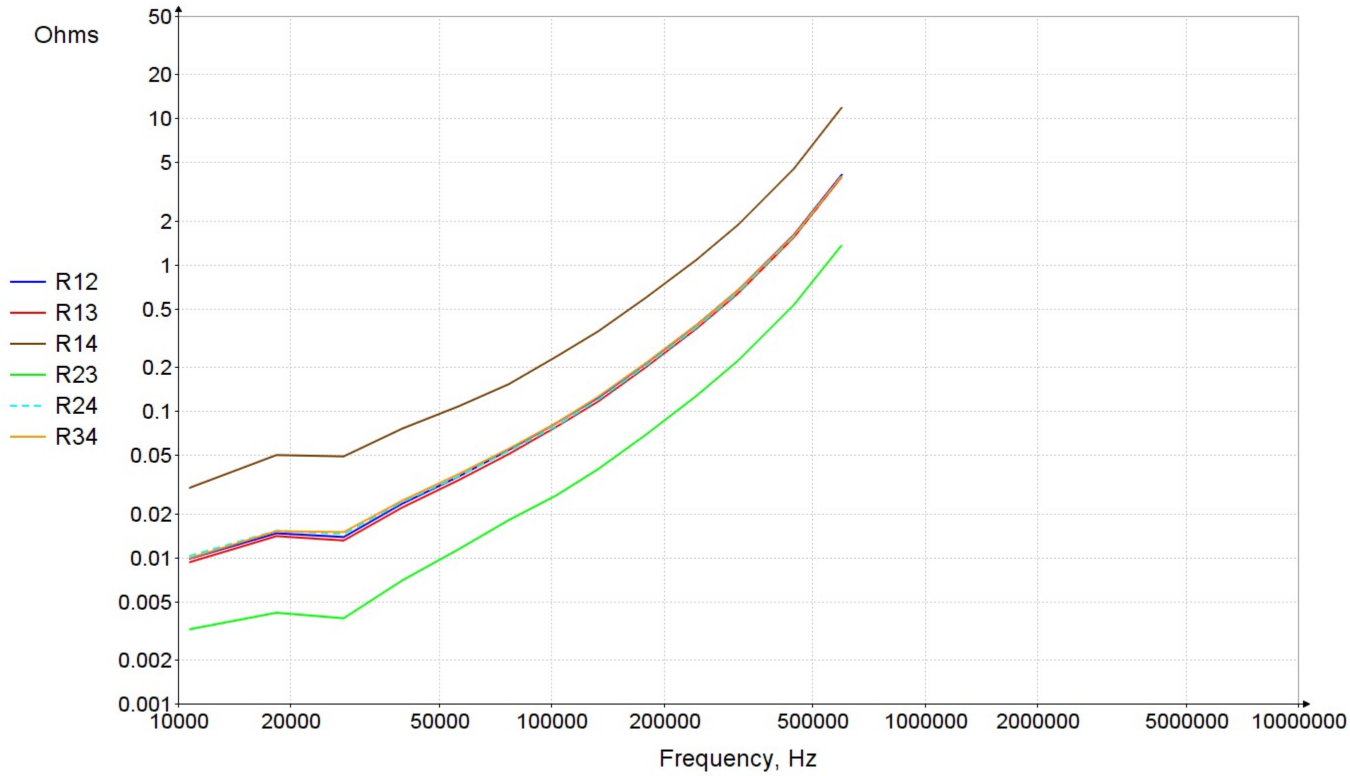


Fig. 4. Measured mutual resistances.

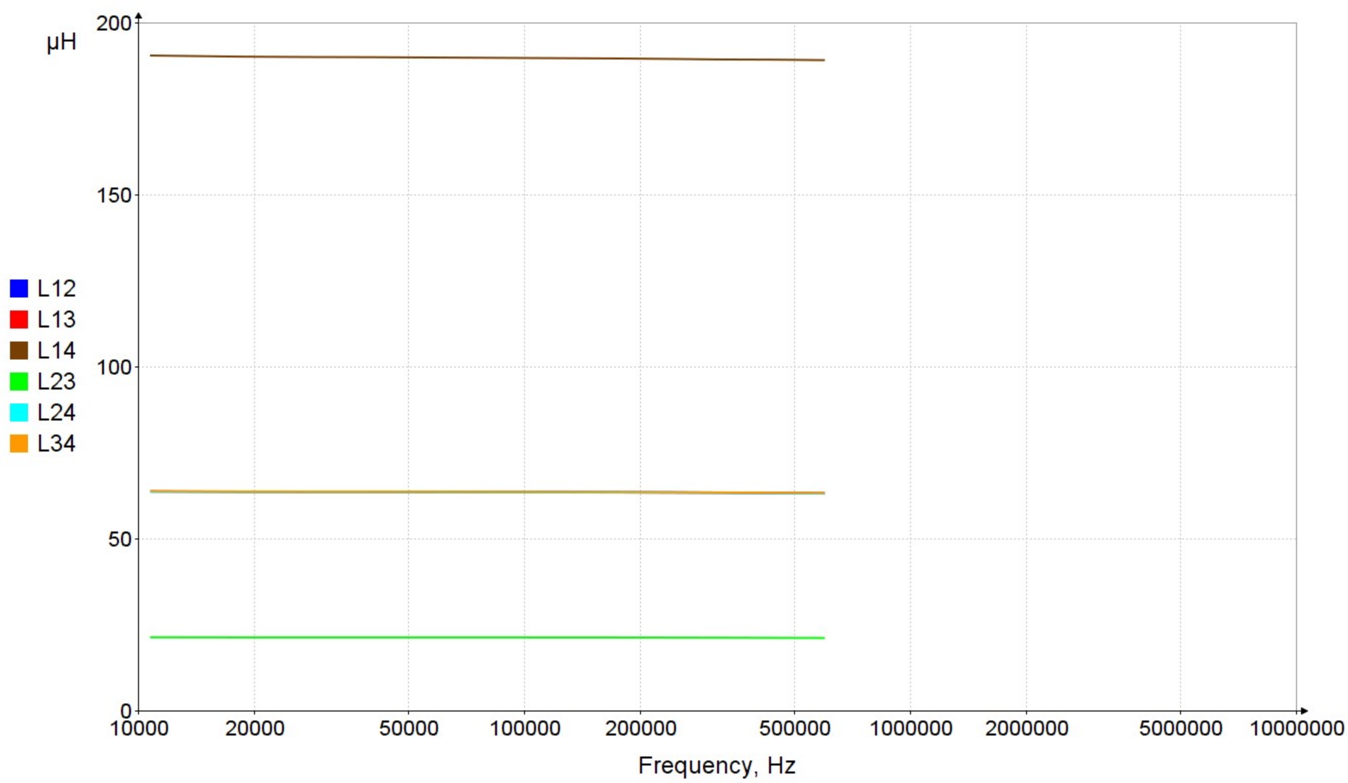


Fig. 5. Measured mutual inductances.

The imported simulation data is made available for subsequent calculation through functions that are indexed by the frequency of the matrix data.

$$RF(f) := \begin{bmatrix} R11_f \\ R22_f \\ R33_f \\ R44_f \\ R12_f \\ R13_f \\ R14_f \\ R23_f \\ R24_f \\ R34_f \end{bmatrix} \qquad LF(f) := \begin{bmatrix} L11_f \\ L22_f \\ L33_f \\ L44_f \\ L12_f \\ L13_f \\ L14_f \\ L23_f \\ L24_f \\ L34_f \end{bmatrix}$$

We begin the process of modeling the transformer by defining the voltages and currents as shown in Fig. 6.

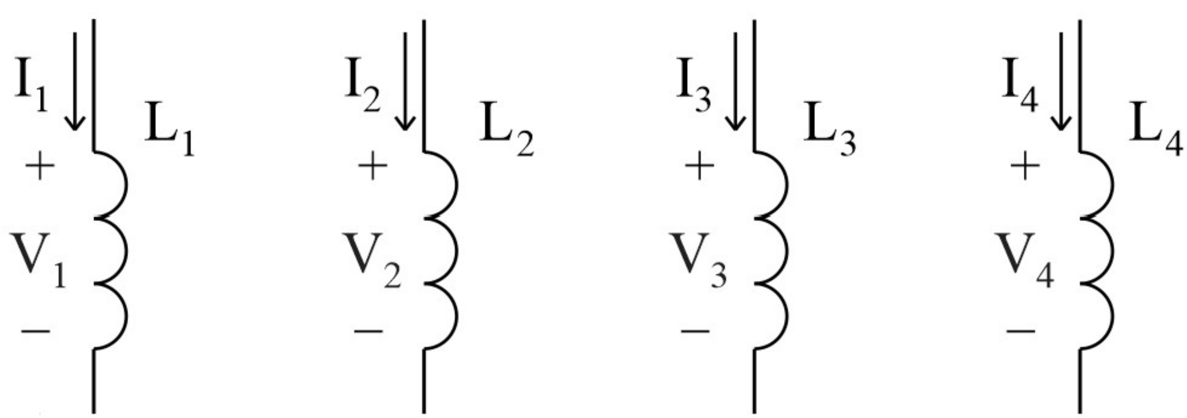


Fig. 6. Voltages and currents

As with any four-port network, the transformer voltages and currents can be described in the frequency domain in terms of self and mutual impedances.

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 + Z_{13} \cdot I_3 + Z_{14} \cdot I_3 \tag{1}$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2 + Z_{23} \cdot I_3 + Z_{24} \cdot I_3 \tag{2}$$

$$V_3 = Z_{31} \cdot I_1 + Z_{32} \cdot I_2 + Z_{33} \cdot I_3 + Z_{34} \cdot I_3 \tag{3}$$

$$V_4 = Z_{41} \cdot I_1 + Z_{42} \cdot I_2 + Z_{43} \cdot I_3 + Z_{44} \cdot I_4 \tag{4}$$

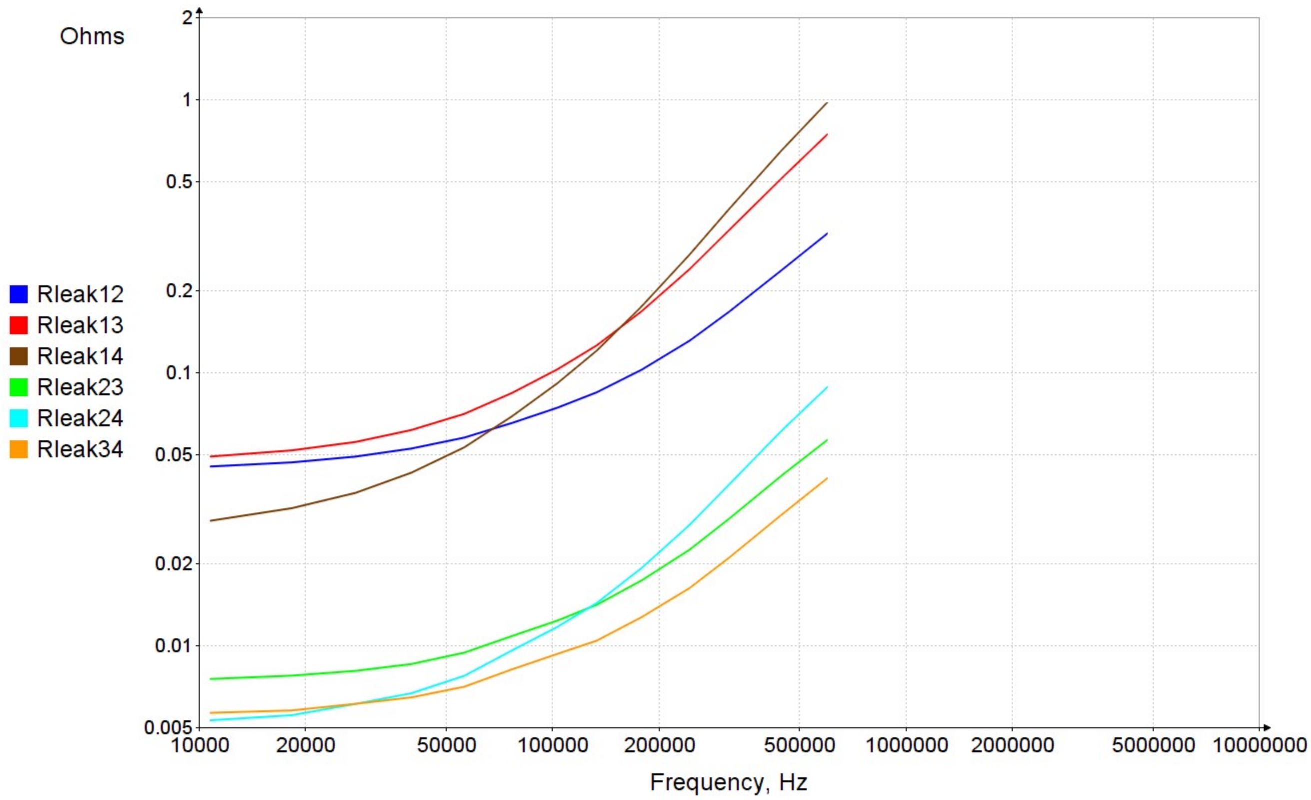


Fig. 7. Measured Leakage resistances.

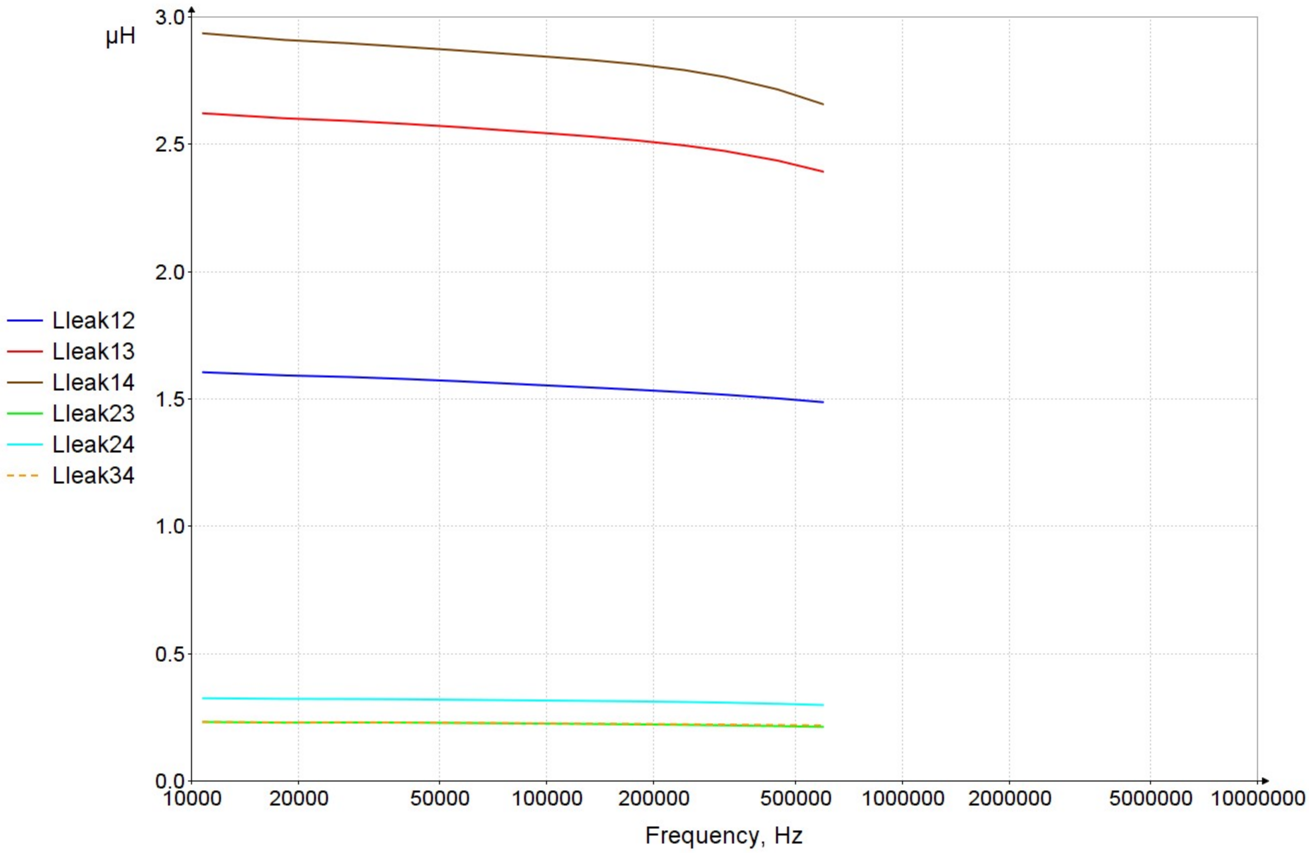
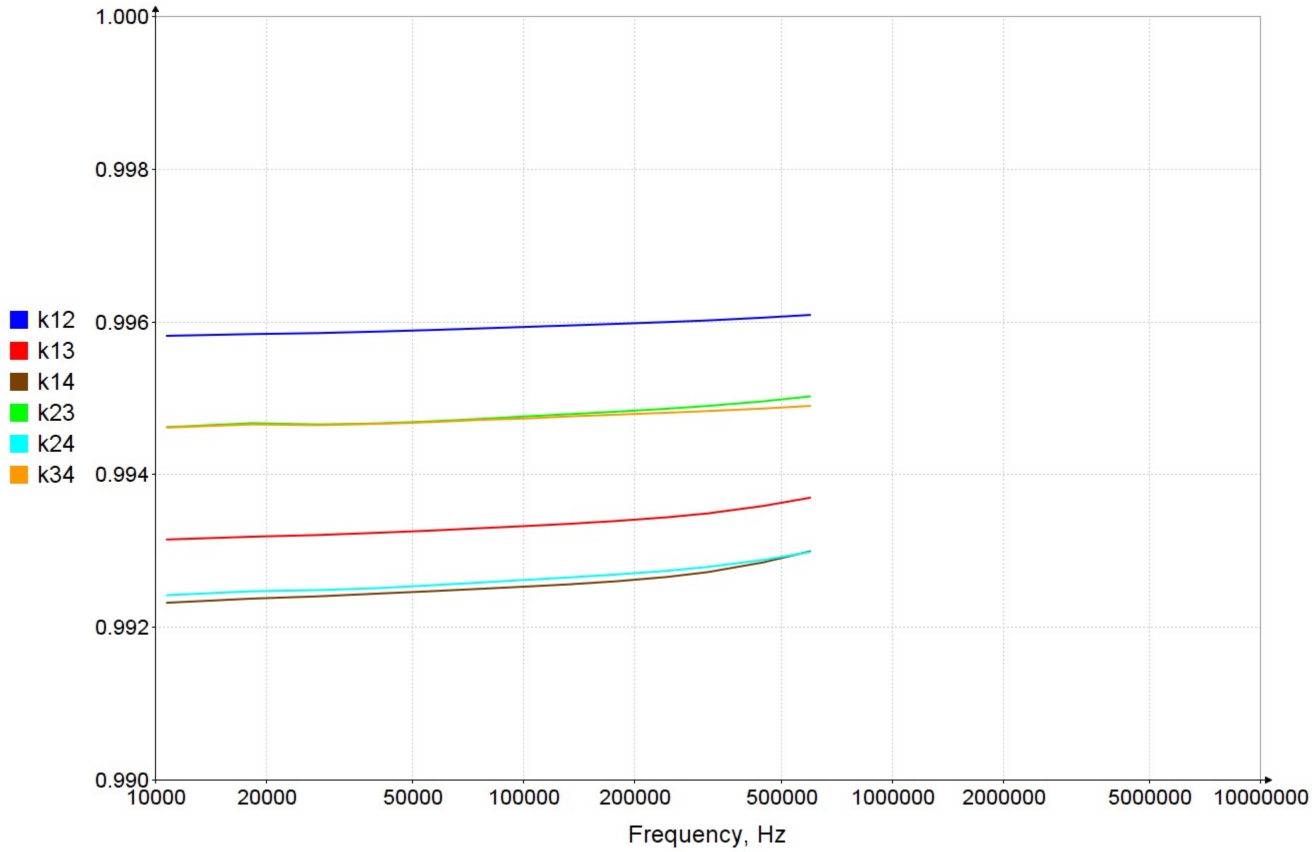


Fig. 8. Leakage inductances.

Mutual inductance couplings

$$k12 := \overrightarrow{\frac{L12}{\sqrt{L11 \cdot L22}}} \qquad k13 := \overrightarrow{\frac{L13}{\sqrt{L11 \cdot L33}}} \qquad k14 := \overrightarrow{\frac{L14}{\sqrt{L11 \cdot L44}}}$$

$$k23 := \overrightarrow{\frac{L23}{\sqrt{L22 \cdot L33}}} \qquad k24 := \overrightarrow{\frac{L24}{\sqrt{L22 \cdot L44}}} \qquad k34 := \overrightarrow{\frac{L34}{\sqrt{L33 \cdot L44}}}$$



Mutual Resistance couplings

$$kr12 := \overrightarrow{\frac{R12}{\sqrt{R11 \cdot R22}}} \qquad kr13 := \overrightarrow{\frac{R13}{\sqrt{R11 \cdot R33}}} \qquad kr14 := \overrightarrow{\frac{R14}{\sqrt{R11 \cdot R44}}}$$

$$kr23 := \overrightarrow{\frac{R23}{\sqrt{R22 \cdot R33}}} \qquad kr24 := \overrightarrow{\frac{R24}{\sqrt{R22 \cdot R44}}} \qquad kr34 := \overrightarrow{\frac{R34}{\sqrt{R33 \cdot R44}}}$$

Mutual Resistance Function

$$KR(f) := \begin{bmatrix} kr12_f \\ kr13_f \\ kr14_f \\ kr23_f \\ kr24_f \\ kr34_f \end{bmatrix}$$

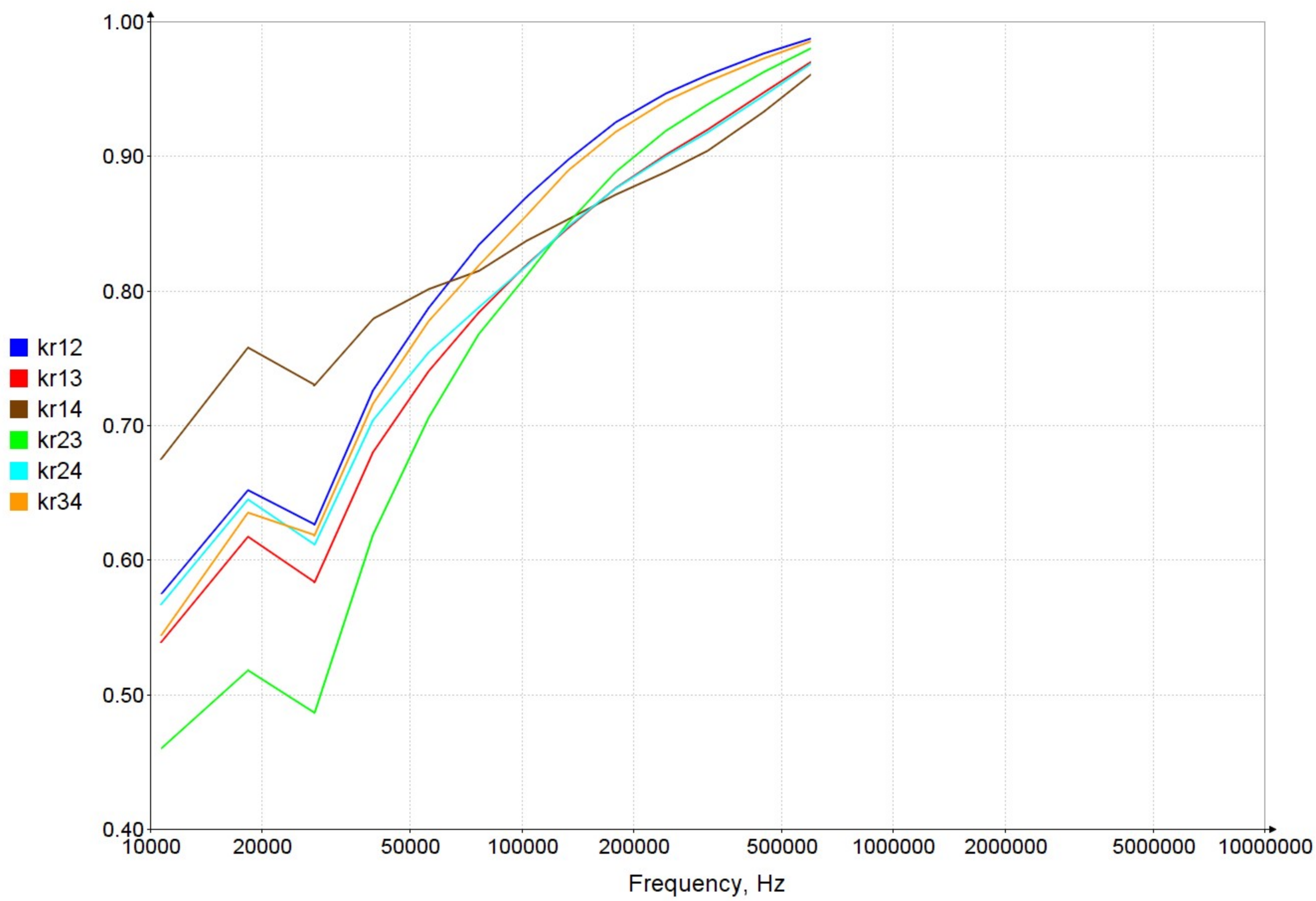
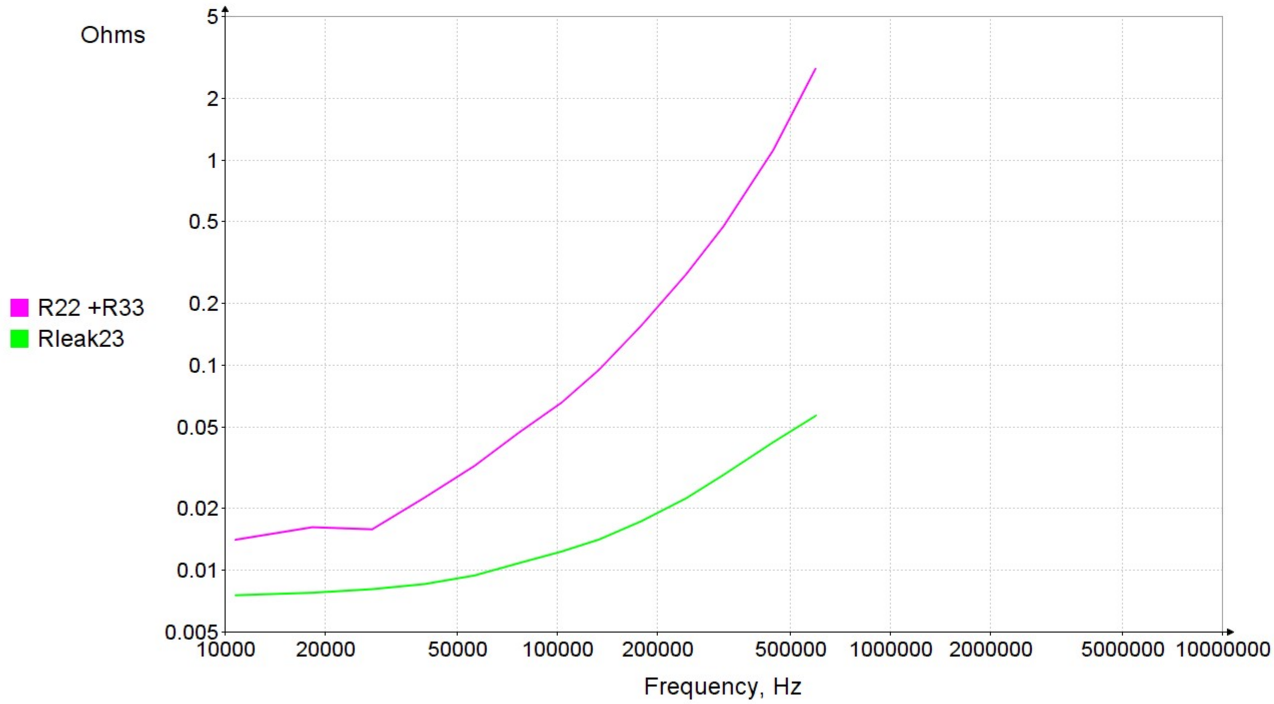


Fig. 9. Mutual Resistance Couplings.



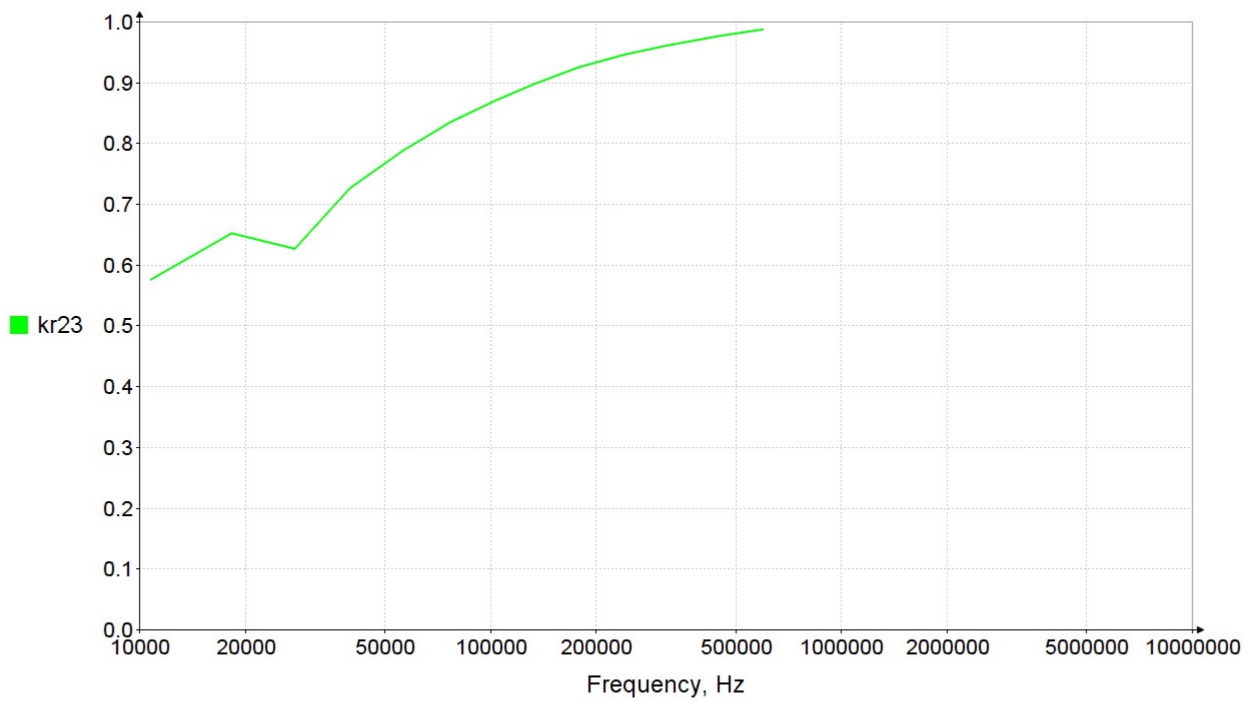
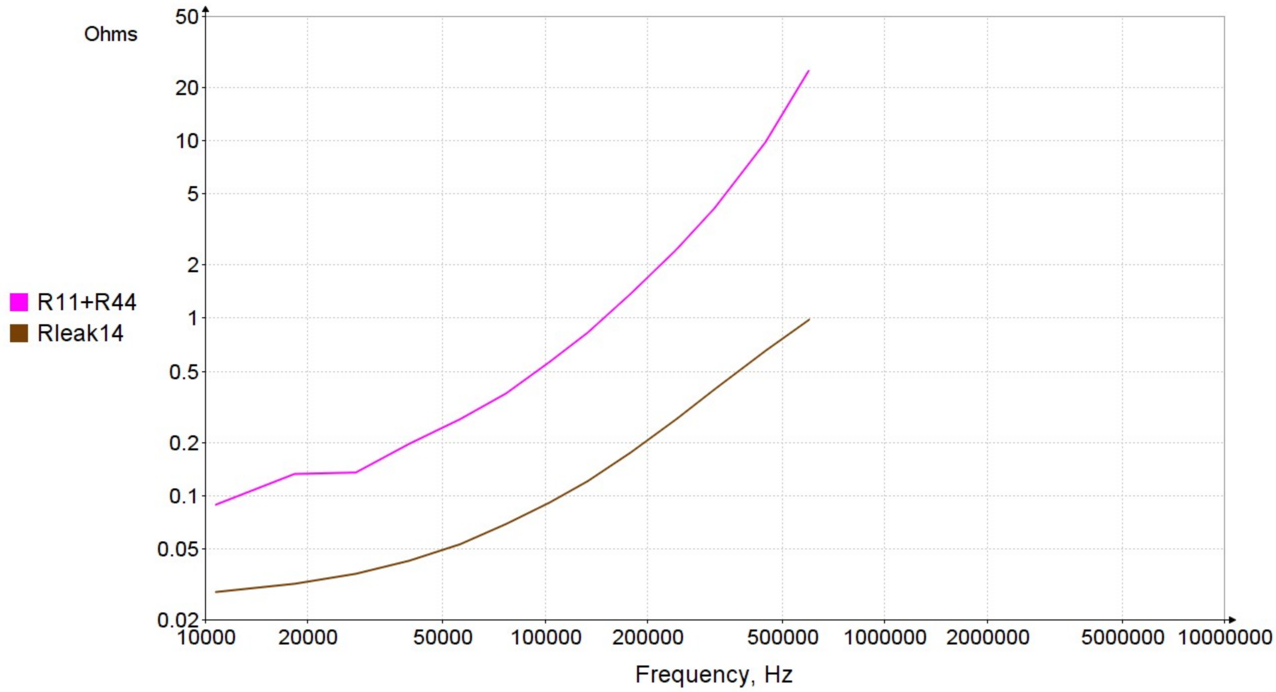


Fig. 10. Comparison between sum of self and reflected self resistances for high positive mutual resistance.

There is a significant reduction of the ac resistance due to the mutual resistance between these adjacent windings.



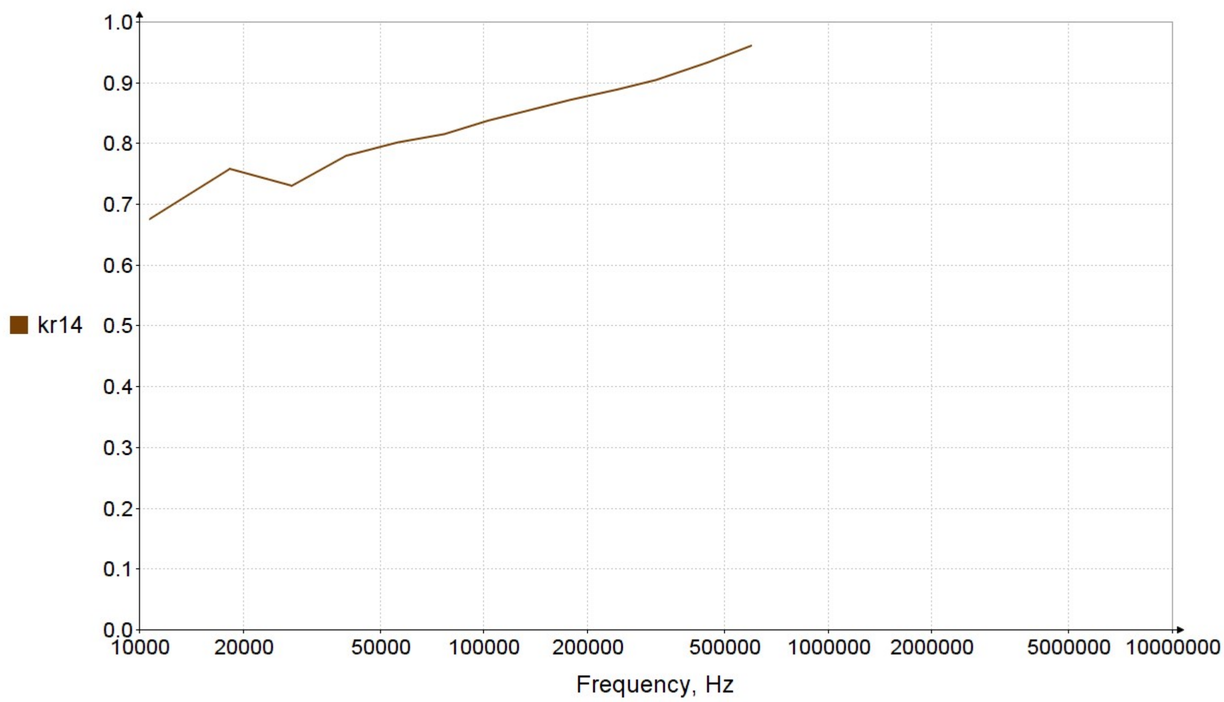


Fig. 11. Comparison between sum of self and reflected self resistances for lower mutual resistance.

These coils are not adjacent, and the resulting lower mutual resistance produces less reduction of the ac resistance compared to Fig. 10.

The impedance matrix description of the transformer can be approximated with the circuit model shown below in Fig. 12 as explained in [1]. For each winding, there is a resistor representing the dc resistance of that winding and a main inductor representing the maximum low-frequency inductance for that winding. The main inductor and the dc resistance for each winding are connected in series between the electrical terminals of that winding. There is also a set of auxiliary circuits for each winding that is shown in a row to the left of the winding. Each auxiliary circuit consists of an auxiliary inductor that is connected in parallel with an auxiliary resistor. The bottom terminals of all of the inductors in each set are connected together to prevent floating nodes, which are not allowed in circuit simulators.

The schematic diagram shows two auxiliary circuits for each winding, but the model could be extended to include more auxiliary circuits. Increasing the number of auxiliary circuits increases the frequency range over which the skin effect can be modeled.

The main inductors are coupled to each other and to each of the auxiliary inductors. The auxiliary inductors are not coupled to each other. It is, of course, impossible to construct a magnetic device in which a set of uncoupled windings are all coupled some other winding. This arrangement is useful as a model, however, and it is possible to describe it mathematically, and to model it in circuit simulators.

The model has one more degree of freedom than is necessary for each auxiliary inductors, so the inductances of the auxiliary inductors in each set are assigned a value equal to the inductance of the main winding associated with that set.

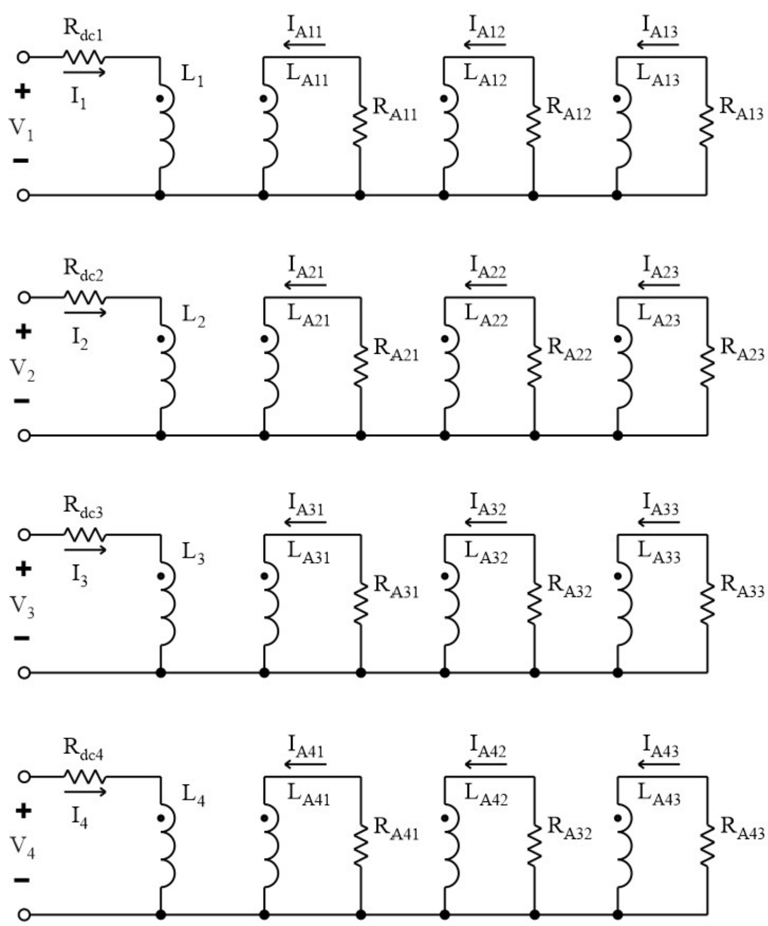


Figure 12. Schematic diagram of transformer circuit model.

We now define several variables and matrices that will be used in an equation that describes the circuit of Fig. 12.

The total number of auxiliary circuits for each winding is designated as r, which is 3 in Fig. 12. The counter variable κ indicates the κth auxiliary circuit, and it ranges from 1 to r.

$N := 4$ $\qquad r := 3$ $\qquad Naux := N \cdot r = 12$ $\qquad Nk := N^2 \cdot r = 48$

The value of each auxiliary inductor is set equal to the value of the corresponding main winding.

$$L_A := \begin{array}{|l} row \leftarrow 0 \\ \text{for } n \in 1..N \\ \quad \begin{array}{|l} \text{for } a \in 1..r \\ \quad \begin{array}{|l} row \leftarrow row + 1 \\ L_{row} \leftarrow Lb_{n,n} \end{array} \end{array} \\ L \end{array}$$

R_A contains the initial guess values of the auxiliary resistors for solve block.

k_A contains the initial guess values of the coupling coefficients between the auxiliary circuits and the main windings.

$$R_A := \begin{bmatrix} 1000 \\ 100 \\ 10 \\ 1000 \\ 100 \\ 10 \\ 1000 \\ 100 \\ 10 \\ 1000 \\ 100 \\ 10 \end{bmatrix} \cdot \Omega$$

$$k_A := \begin{array}{|l} \text{for } n \in 1..Nk \\ \quad \begin{array}{|l} K_n \leftarrow 0.01 \end{array} \\ K \end{array}$$

The mutual inductance between L_1 and L_2 is designated M_{12}.

The elements of M are calculated as shown below.

$$M\left(k_A\right) := \left\| \begin{array}{l} a \leftarrow 1 \\ Cols \leftarrow Naux \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1..Cols \\ \quad \left\| \begin{array}{l} M_{row,col} \leftarrow k_{A_a} \cdot \sqrt{L_{A_{col}} \cdot Lb_{row,row}} \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ M \end{array} \right.$$

The value of each auxiliary inductor is set equal to the value of the corresponding main winding.

$$L_A := \left\| \begin{array}{l} row \leftarrow 0 \\ \text{for } n \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } a \in 1..r \\ \quad \left\| \begin{array}{l} row \leftarrow row+1 \\ L_{row} \leftarrow Lb_{n,n} \end{array} \right. \end{array} \right. \\ L \end{array} \right.$$

Functions to define the G and B matrices described in [1]. G and B are needed to find the impedances of the transformer equivalent circuit shown in Fig. 12.

$$G\left(f, L_A, R_A\right) := \left\| \begin{array}{l} G_A \leftarrow \overrightarrow{\left(\dfrac{R_A}{{R_A}^2 + \left(\omega_f\right)^2 \cdot {L_A}^2}\right)} \\ X \leftarrow \dfrac{\text{identity}\left(Naux\right)}{\boldsymbol{\Omega}} \\ \text{for } n \in 1..Naux \\ \quad \left\| X_{n,n} \leftarrow G_{A_n} \right. \\ X \end{array} \right.$$

$$B\left(f, L_A, R_A\right) := \left\| \begin{array}{l} B_A \leftarrow \overrightarrow{\left(\dfrac{L_A}{{R_A}^2 + \left(\omega_f\right)^2 \cdot {L_A}^2}\right)} \\ X \leftarrow \text{identity}\left(Naux\right) \cdot \dfrac{\mathbf{s}}{\boldsymbol{\Omega}} \\ \text{for } n \in 1..Naux \\ \quad \left\| X_{n,n} \leftarrow B_{A_n} \right. \\ X \end{array} \right.$$

$\text{rows}\left(G\left(1, L_A, R_A\right)\right) = 12$

$\text{cols}\left(G\left(1, L_A, R_A\right)\right) = 12$

$\text{rows}\left(B\left(1, L_A, R_A\right)\right) = 12$

$\text{cols}\left(B\left(1, L_A, R_A\right)\right) = 12$

Function to calculate the equivalent circuit self and mutual resistances.

$$R_{eq}\left(R_A, k_A, f\right) := \left\| \begin{array}{l} R \leftarrow Rb + \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot G\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \begin{bmatrix} R_{1,1} \\ R_{2,2} \\ R_{3,3} \\ R_{4,4} \\ R_{1,2} \\ R_{1,3} \\ R_{1,4} \\ R_{2,3} \\ R_{2,4} \\ R_{3,4} \end{bmatrix} \end{array} \right.$$

$$\mathrm{rows}\left(R_{eq}\left(R_A, k_A, 1\right)\right) = 10$$

$$\mathrm{cols}\left(R_{eq}\left(R_A, k_A, 1\right)\right) = 1$$

Function to calculate self and mutual inductances of the the equivalent circuit

$$L_{eq}\left(R_A, k_A, f\right) := \left\| \begin{array}{l} L \leftarrow Lb - \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot B\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \begin{bmatrix} L_{1,1} \\ L_{2,2} \\ L_{3,3} \\ L_{4,4} \\ L_{1,2} \\ L_{1,3} \\ L_{1,4} \\ L_{2,3} \\ L_{2,4} \\ L_{3,4} \end{bmatrix} \end{array} \right.$$

Function to calculate the equivalent circuit self and mutual impedances.

$$Z_{eq}\left(R_A, k_A, f\right) := R_{eq}\left(R_A, k_A, f\right) + 1\mathrm{j} \cdot \omega_f \cdot L_{eq}\left(R_A, k_A, f\right)$$

Functions to calculate leakage impedances based on (5).

$$Z_{leak\_mn} = Z_{mm} - \frac{Z_{mn}{}^2}{Z_{nn}} \qquad (5)$$

$$Rleak12EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_5\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_2}\right)$$

$$Rleak13EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_6\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_3}\right)$$

$$Rleak14EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_7\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

$$Rleak23EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_2 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_8\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_3}\right)$$

$$Rleak24EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_2 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_9\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

$$Rleak34EQ\left(R_A, k_A, f\right) := \mathrm{Re}\left(Z_{eq}\left(R_A, k_A, f\right)_3 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_{10}\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

$$Lleak12EQ\left(R_A, k_A, f\right) := \frac{1}{\omega_f} \cdot \mathrm{Im}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_5\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_2}\right)$$

$$Lleak13EQ\left(R_A, k_A, f\right) := \frac{1}{\omega_f} \cdot \mathrm{Im}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_6\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_3}\right)$$

$$Lleak14EQ\left(R_A, k_A, f\right) := \frac{1}{\omega_f} \cdot \mathrm{Im}\left(Z_{eq}\left(R_A, k_A, f\right)_1 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_7\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

$$Lleak23EQ\left(R_A, k_A, f\right) := \frac{1}{\omega_f} \cdot \mathrm{Im}\left(Z_{eq}\left(R_A, k_A, f\right)_2 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_8\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_3}\right)$$

$$Lleak24EQ\left(R_A, k_A, f\right) := \frac{1}{\omega_f} \cdot \mathrm{Im}\left(Z_{eq}\left(R_A, k_A, f\right)_2 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_9\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

$$Lleak34EQ\left(R_A, k_A, f\right) := \frac{1}{\omega_f} \cdot \mathrm{Im}\left(Z_{eq}\left(R_A, k_A, f\right)_3 - \frac{\left(Z_{eq}\left(R_A, k_A, f\right)_{10}\right)^2}{Z_{eq}\left(R_A, k_A, f\right)_4}\right)$$

Function to calculate the mutual resistance couplings of the the equivalent circuit

$$kreq\left(R_A, k_A, f\right) := \left\| \begin{array}{l} R \leftarrow Rb + \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot G\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \begin{bmatrix} \dfrac{R_{1,2}}{\sqrt{R_{1,1} \cdot R_{2,2}}} \\ \dfrac{R_{1,3}}{\sqrt{R_{1,1} \cdot R_{3,3}}} \\ \dfrac{R_{1,4}}{\sqrt{R_{1,1} \cdot R_{4,4}}} \\ \dfrac{R_{2,3}}{\sqrt{R_{2,2} \cdot R_{3,3}}} \\ \dfrac{R_{2,4}}{\sqrt{R_{2,2} \cdot R_{4,4}}} \\ \dfrac{R_{3,4}}{\sqrt{R_{3,3} \cdot R_{4,4}}} \end{bmatrix} \end{array} \right.$$

Define error functions base on mutual resistance coupling, resistance and inductance

$$Error\_KR\left(R_A, k_A, f\right) := \left\| \begin{array}{l} kr \leftarrow KR(f) \\ kreq\left(R_A, k_A, f\right) - kr \end{array} \right.$$

$$Error\_R\left(R_A, k_A, f\right) := \left\| \begin{array}{l} Rf \leftarrow RF(f) \\ D \leftarrow R_{eq}\left(R_A, k_A, f\right) - Rf \\ \overrightarrow{\dfrac{D}{Rf}} \end{array} \right.$$

$$Error\_L\left(R_A, k_A, f\right) := \left\| \begin{array}{l} Lf \leftarrow LF(f) \\ D \leftarrow L_{eq}\left(R_A, k_A, f\right) - Lf \\ \overrightarrow{\dfrac{D}{Lf}} \end{array} \right.$$

Vectors of zeros used in the Minerr block.

$$Z := \left\| \begin{array}{l} Z_{\frac{N^2+N}{2}} \leftarrow 0 \\ Z \end{array} \right. \qquad Zk := \left\| \begin{array}{l} Z_{\frac{N^2-N}{2}} \leftarrow 0 \\ Z \end{array} \right.$$

Define a function for calculating the coupling matrix of the equivalent circuit.

$$K_{mod}(k_A) := \left\| \begin{array}{l} K \leftarrow \text{identity}(N+Naux) \\ \text{for } m \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } n \in 1..N \\ \quad \| \; K_{m,n} \leftarrow Kb_{m,n} \end{array} \right. \\ a \leftarrow 1 \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in N+1..N+Naux \\ \quad \left\| \begin{array}{l} K_{row,col} \leftarrow k_{A_a} \\ K_{col,row} \leftarrow k_{A_a} \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ K \end{array} \right.$$

Define function for computing the eigenvalues of the model coupling matrix.

$$EigenTest(k_A) := \left\| \begin{array}{l} EV \leftarrow \text{eigenvals}(K_{mod}(k_A)) \\ EV \end{array} \right.$$

$$EigenTest(k_A) = \begin{bmatrix} 3.983045 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0.998391 \\ 0.009303 \\ 0.005569 \\ 0.003692 \end{bmatrix}$$

Define function for determining the if there is a negative eigenvalue for the model coupling matrix. This helps the solver find solutions that are physically realizable, which requires all of the eigenvalues to be positive [2]. The 1000000 multiplier gives it a high weight in the solver.

$$EigenSign(k_A) := \left\| \begin{array}{l} ET \leftarrow min(EigenTest(k_A)) \\ \dfrac{10000000 \cdot ET}{|ET| + 10^{-10}} \end{array} \right. \qquad EigenSign(k_A) = 10 \cdot 10^{6}$$

Error Weighting $c := 100$ $d := 10$ $e := 1$

$d \cdot Error\_R(R_A, k_A, 1) = Z$ $d \cdot Error\_R(R_A, k_A, 5) = Z$ $d \cdot Error\_R(R_A, k_A, 10) = Z$

$d \cdot Error\_R(R_A, k_A, 2) = Z$ $d \cdot Error\_R(R_A, k_A, 6) = Z$ $d \cdot Error\_R(R_A, k_A, 11) = Z$

$d \cdot Error\_R(R_A, k_A, 3) = Z$ $d \cdot Error\_R(R_A, k_A, 7) = Z$ $d \cdot Error\_R(R_A, k_A, 12) = Z$

$d \cdot Error\_R(R_A, k_A, 4) = Z$ $d \cdot Error\_R(R_A, k_A, 8) = Z$ $d \cdot Error\_R(R_A, k_A, 13) = Z$

$c \cdot Error\_L(R_A, k_A, 1) = Z$ $c \cdot Error\_L(R_A, k_A, 5) = Z$ $c \cdot Error\_L(R_A, k_A, 10) = Z$

$c \cdot Error\_L(R_A, k_A, 2) = Z$ $c \cdot Error\_L(R_A, k_A, 6) = Z$ $c \cdot Error\_L(R_A, k_A, 11) = Z$

$c \cdot Error\_L(R_A, k_A, 3) = Z$ $c \cdot Error\_L(R_A, k_A, 7) = Z$ $c \cdot Error\_L(R_A, k_A, 12) = Z$

$c \cdot Error\_L(R_A, k_A, 4) = Z$ $c \cdot Error\_L(R_A, k_A, 8) = Z$ $c \cdot Error\_L(R_A, k_A, 13) = Z$

Guess Values

$e \cdot Error\_KR(R_A, k_A, 1) = Zk$ $e \cdot Error\_KR(R_A, k_A, 5) = Zk$ $e \cdot Error\_KR(R_A, k_A, 9) = Zk$

$e \cdot Error\_KR(R_A, k_A, 2) = Zk$ $e \cdot Error\_KR(R_A, k_A, 6) = Zk$ $e \cdot Error\_KR(R_A, k_A, 10) = Zk$

$e \cdot Error\_KR(R_A, k_A, 3) = Zk$ $e \cdot Error\_KR(R_A, k_A, 7) = Zk$ $e \cdot Error\_KR(R_A, k_A, 11) = Zk$

$e \cdot Error\_KR(R_A, k_A, 4) = Zk$ $e \cdot Error\_KR(R_A, k_A, 8) = Zk$ $e \cdot Error\_KR(R_A, k_A, 13) = Zk$

traints

Constraints on coupling coefficients and resistors prevent inappropriate component values.

Cons

$-1<k_{A_1}<1$ $\quad -1<k_{A_9}<1$ $\quad -1<k_{A_{17}}<1$ $\quad -1<k_{A_{25}}<1$ $\quad -1<k_{A_{33}}<1$ $\quad -1<k_{A_{41}}<1$

$-1<k_{A_2}<1$ $\quad -1<k_{A_{10}}<1$ $\quad -1<k_{A_{18}}<1$ $\quad -1<k_{A_{26}}<1$ $\quad -1<k_{A_{34}}<1$ $\quad -1<k_{A_{42}}<1$

$-1<k_{A_3}<1$ $\quad -1<k_{A_{11}}<1$ $\quad -1<k_{A_{19}}<1$ $\quad -1<k_{A_{27}}<1$ $\quad -1<k_{A_{35}}<1$ $\quad -1<k_{A_{43}}<1$

$-1<k_{A_4}<1$ $\quad -1<k_{A_{12}}<1$ $\quad -1<k_{A_{20}}<1$ $\quad -1<k_{A_{28}}<1$ $\quad -1<k_{A_{36}}<1$ $\quad -1<k_{A_{44}}<1$

$-1<k_{A_5}<1$ $\quad -1<k_{A_{13}}<1$ $\quad -1<k_{A_{21}}<1$ $\quad -1<k_{A_{29}}<1$ $\quad -1<k_{A_{37}}<1$ $\quad -1<k_{A_{45}}<1$

$-1<k_{A_6}<1$ $\quad -1<k_{A_{14}}<1$ $\quad -1<k_{A_{22}}<1$ $\quad -1<k_{A_{30}}<1$ $\quad -1<k_{A_{38}}<1$ $\quad -1<k_{A_{46}}<1$

$-1<k_{A_7}<1$ $\quad -1<k_{A_{15}}<1$ $\quad -1<k_{A_{23}}<1$ $\quad -1<k_{A_{31}}<1$ $\quad -1<k_{A_{39}}<1$ $\quad -1<k_{A_{47}}<1$

$-1<k_{A_8}<1$ $\quad -1<k_{A_{16}}<1$ $\quad -1<k_{A_{24}}<1$ $\quad -1<k_{A_{32}}<1$ $\quad -1<k_{A_{40}}<1$ $\quad -1<k_{A_{48}}<1$

$10^4 \cdot R_{A_1} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_4} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_7} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_{10}} > 10^5 \cdot \Omega$

$10^4 \cdot R_{A_2} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_5} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_8} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_{11}} > 10^5 \cdot \Omega$

$10^4 \cdot R_{A_3} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_6} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_9} > 10^5 \cdot \Omega$ $\quad 10^4 \cdot R_{A_{12}} > 10^5 \cdot \Omega$

$EigenSign\left(k_A\right) > 10000000$

Solver

$$\begin{bmatrix} R_A \\ k_A \end{bmatrix} := \mathbf{Minerr}\left(R_A, k_A\right)$$

$ERR = ?$

$$R_A = \begin{bmatrix} 4.7131 \cdot 10^3 \\ 10.0000 \\ 10.0000 \\ 419.6790 \\ 512.4565 \\ 10.0000 \\ 418.5462 \\ 513.5535 \\ 10.0000 \\ 4.6899 \cdot 10^3 \\ 10.0000 \\ 10.0000 \end{bmatrix} \Omega \qquad Kb = \begin{bmatrix} 1.00000 & 0.99581 & 0.99313 & 0.99231 \\ 0.99581 & 1.00000 & 0.99462 & 0.99240 \\ 0.99313 & 0.99462 & 1.00000 & 0.99461 \\ 0.99231 & 0.99240 & 0.99461 & 1.00000 \end{bmatrix}$$

$$\text{eigenvals}\left(K_{mod}\left(k_A\right)\right)=\begin{bmatrix}4.069991\\1.007516\\1.003385\\1.001120\\1.000000\\1.000000\\1.000000\\1.000000\\1.000000\\1.000000\\1.000000\\1.000000\\0.911537\\0.002979\\0.001736\\0.001736\end{bmatrix}$$

If negative eigenvalues values are present then the realizability criterion is violated [2].

Computed auxiliary couplings

$$k_{A1}:=\left\|\begin{array}{l}\text{for } h\in 1..24\\ \quad\left\| K_h\leftarrow k_{A_h}\right.\\ K\end{array}\right.$$

$$k_{A2}:=\left\|\begin{array}{l}\text{for } h\in 25..48\\ \quad\left\| K_{h-24}\leftarrow k_{A_h}\right.\\ K\end{array}\right.$$

$$k_{A1}=\begin{bmatrix} 0.101941 \\ 0.018113 \\ 0.047692 \\ 0.112206 \\ 0.085697 \\ 0.005300 \\ 0.115463 \\ 0.087365 \\ 0.003659 \\ 0.105532 \\ 0.014285 \\ 0.043414 \\ 0.119170 \\ 0.044617 \\ 0.031972 \\ 0.098809 \\ 0.114845 \\ -0.000024 \\ 0.100950 \\ 0.115842 \\ 0.008167 \\ 0.080470 \\ 0.022742 \\ 0.017458 \end{bmatrix} \qquad k_{A2}=\begin{bmatrix} 0.110585 \\ 0.010939 \\ 0.028829 \\ 0.068709 \\ 0.125990 \\ 0.001742 \\ 0.067353 \\ 0.130540 \\ -0.011175 \\ 0.117313 \\ 0.041810 \\ 0.026554 \\ 0.133282 \\ 0.029717 \\ 0.025758 \\ 0.060893 \\ 0.102054 \\ -0.007944 \\ 0.055288 \\ 0.096168 \\ -0.002023 \\ 0.156825 \\ 0.033670 \\ 0.049152 \end{bmatrix}$$

Extract resistance and inductance values of the equivalent circuit for plotting.

$$R11eq_f := R_{eq}\left(R_A, k_A, f\right)_1 \qquad L11eq_f := L_{eq}\left(R_A, k_A, f\right)_1$$

$$R22eq_f := R_{eq}\left(R_A, k_A, f\right)_2 \qquad L22eq_f := L_{eq}\left(R_A, k_A, f\right)_2$$

$$R33eq_f := R_{eq}\left(R_A, k_A, f\right)_3 \qquad L33eq_f := L_{eq}\left(R_A, k_A, f\right)_3$$

$$R44eq_f := R_{eq}\left(R_A, k_A, f\right)_4 \qquad L44eq_f := L_{eq}\left(R_A, k_A, f\right)_4$$

$$R12eq_f := R_{eq}\left(R_A, k_A, f\right)_5 \qquad L12eq_f := L_{eq}\left(R_A, k_A, f\right)_5$$

$$R13eq_f := R_{eq}\left(R_A, k_A, f\right)_6 \qquad L13eq_f := L_{eq}\left(R_A, k_A, f\right)_6$$

$$R14eq_f := R_{eq}\left(R_A, k_A, f\right)_7 \qquad L14eq_f := L_{eq}\left(R_A, k_A, f\right)_7$$

$$R23eq_f := R_{eq}\left(R_A, k_A, f\right)_8 \qquad L23eq_f := L_{eq}\left(R_A, k_A, f\right)_8$$

$$R24eq_f := R_{eq}\left(R_A, k_A, f\right)_9 \qquad L24eq_f := L_{eq}\left(R_A, k_A, f\right)_9$$

$$R34eq_f := R_{eq}\left(R_A, k_A, f\right)_{10} \qquad L34eq_f := L_{eq}\left(R_A, k_A, f\right)_{10}$$

Equivalent Circuit mutual resistance couplings

$$kr12eq_f := kreq\left(R_A, k_A, f\right)_1 \qquad kr13eq_f := kreq\left(R_A, k_A, f\right)_2 \qquad kr14eq_f := kreq\left(R_A, k_A, f\right)_3$$

$$kr23eq_f := kreq\left(R_A, k_A, f\right)_4 \qquad kr24eq_f := kreq\left(R_A, k_A, f\right)_5 \qquad kr34eq_f := kreq\left(R_A, k_A, f\right)_6$$

Equivalent circuit leakage resistances and inductances

$$Rleak12eq_f := Rleak12EQ\left(R_A, k_A, f\right) \qquad Lleak12eq_f := Lleak12EQ\left(R_A, k_A, f\right)$$

$$Rleak13eq_f := Rleak13EQ\left(R_A, k_A, f\right) \qquad Lleak13eq_f := Lleak13EQ\left(R_A, k_A, f\right)$$

$$Rleak14eq_f := Rleak14EQ\left(R_A, k_A, f\right) \qquad Lleak14eq_f := Lleak14EQ\left(R_A, k_A, f\right)$$

$$Rleak23eq_f := Rleak23EQ\left(R_A, k_A, f\right) \qquad Lleak23eq_f := Lleak23EQ\left(R_A, k_A, f\right)$$

$$Rleak24eq_f := Rleak24EQ\left(R_A, k_A, f\right) \qquad Lleak24eq_f := Lleak24EQ\left(R_A, k_A, f\right)$$

$$Rleak34eq_f := Rleak34EQ\left(R_A, k_A, f\right) \qquad Lleak34eq_f := Lleak34EQ\left(R_A, k_A, f\right)$$

$$Qleak12_f := \frac{\omega_f \cdot Lleak12_f}{Rleak12_f}$$

$$Qleak13_f := \frac{\omega_f \cdot Lleak13_f}{Rleak13_f}$$

$$Qleak14_f := \frac{\omega_f \cdot Lleak14_f}{Rleak14_f}$$

$$Qleak23_f := \frac{\omega_f \cdot Lleak23_f}{Rleak23_f}$$

$$Qleak24_f := \frac{\omega_f \cdot Lleak24_f}{Rleak24eq_f}$$

$$Qleak34_f := \frac{\omega_f \cdot Lleak34_f}{Rleak34_f}$$

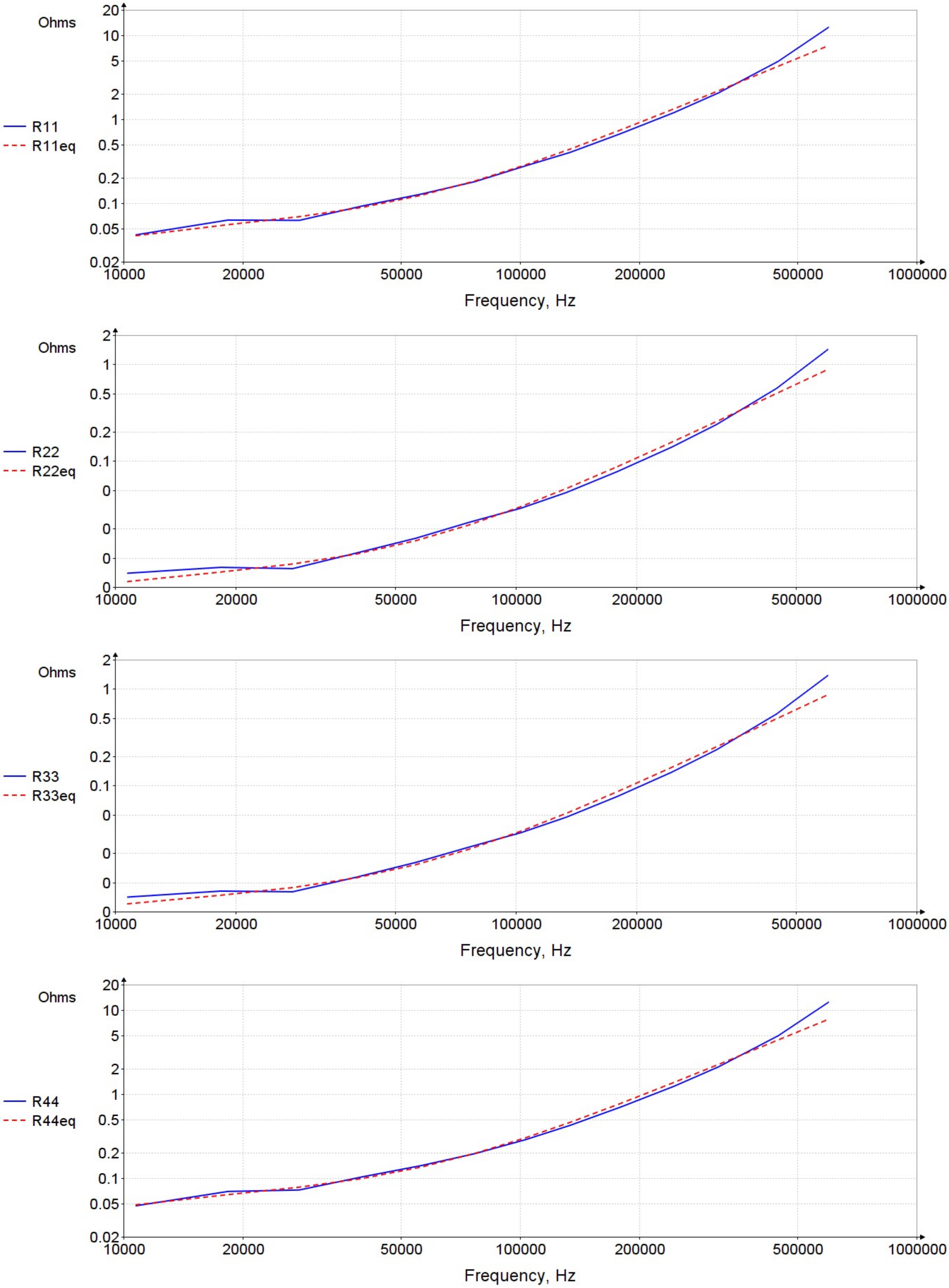


Figure 13. Measured and Equivalent Circuit self resistances.

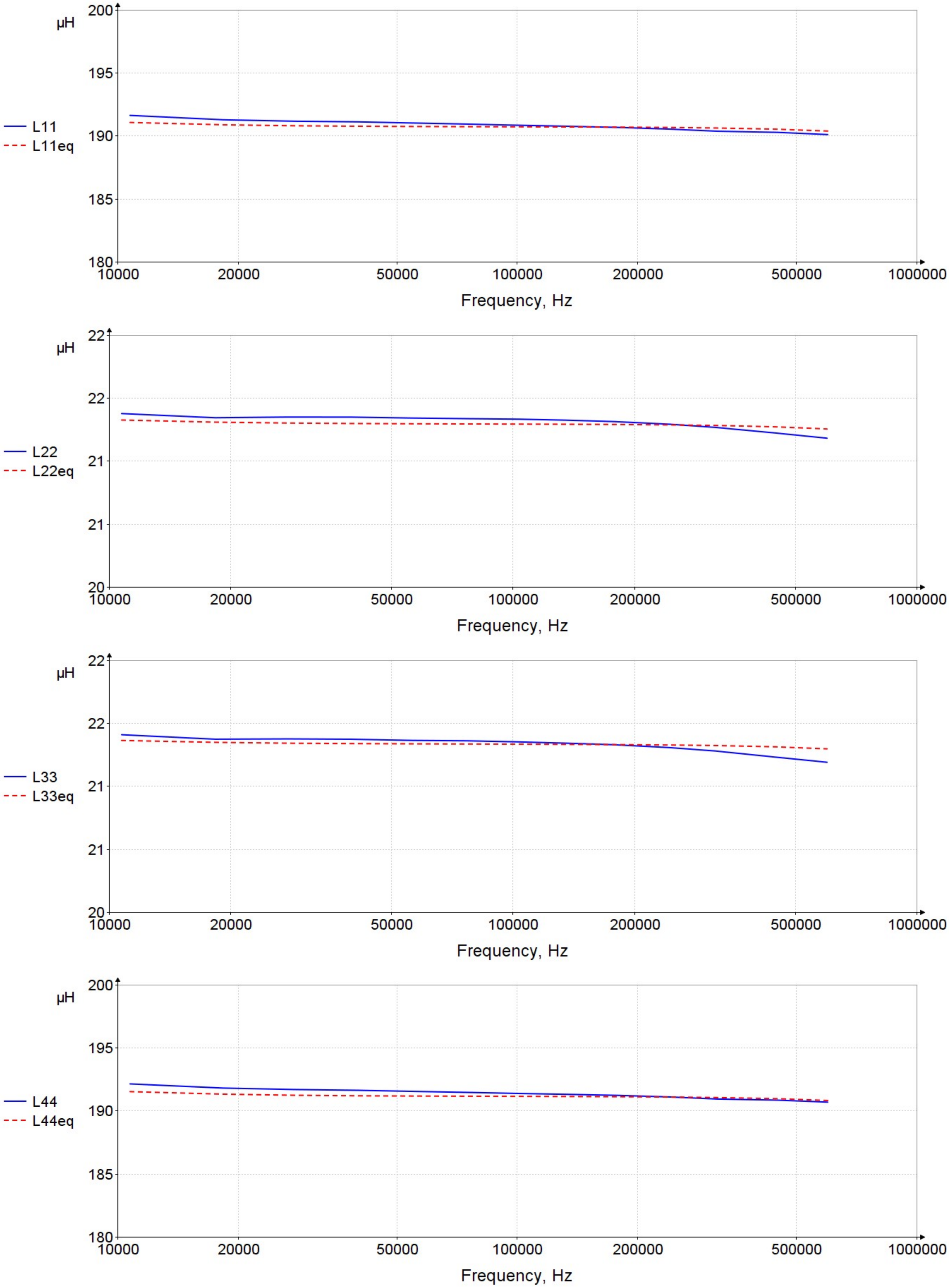


Figure 14. Measured and Equivalent Circuit self inductances.

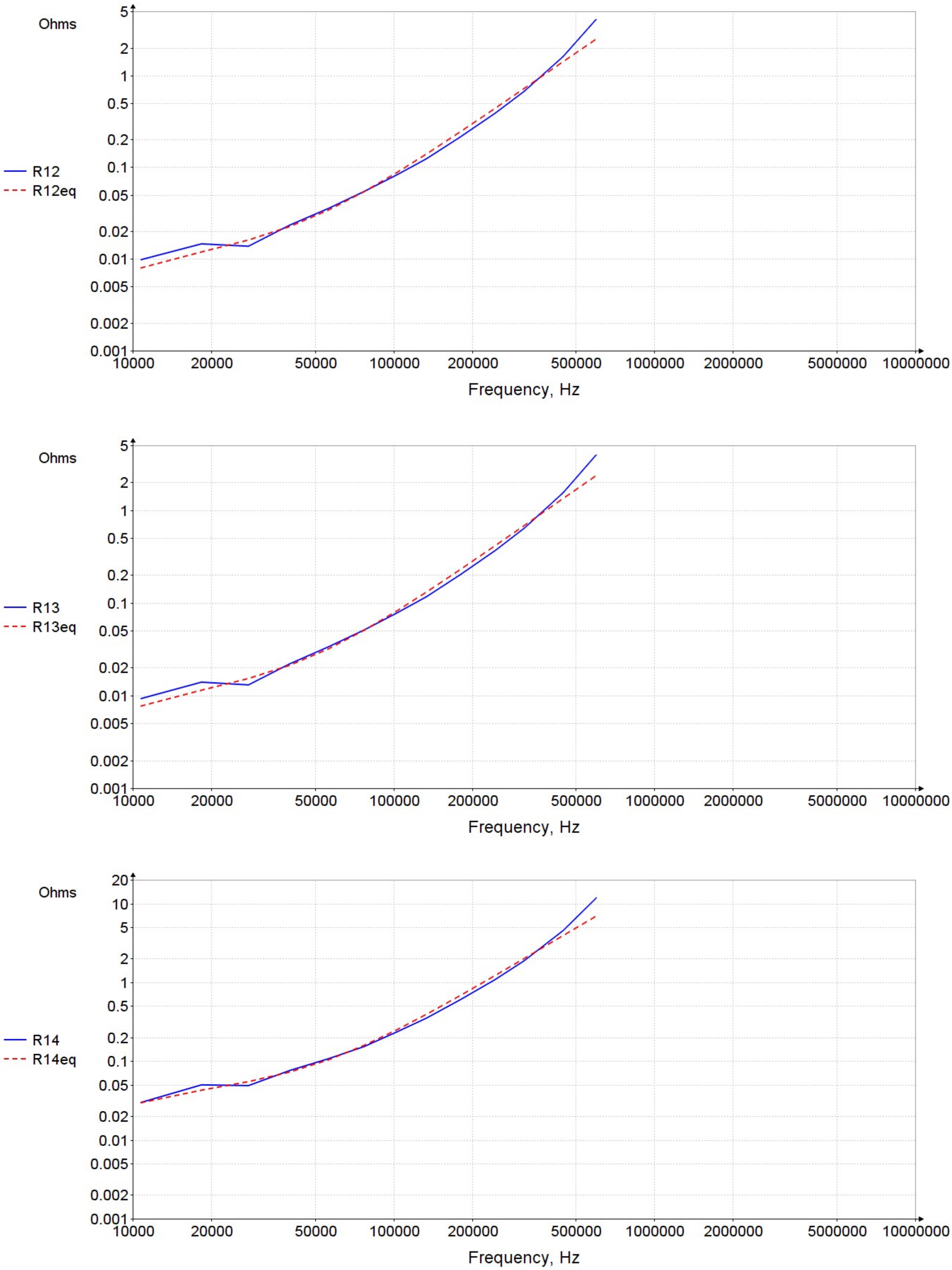


Figure 15a. Measured and Equivalent Circuit mutual resistances.

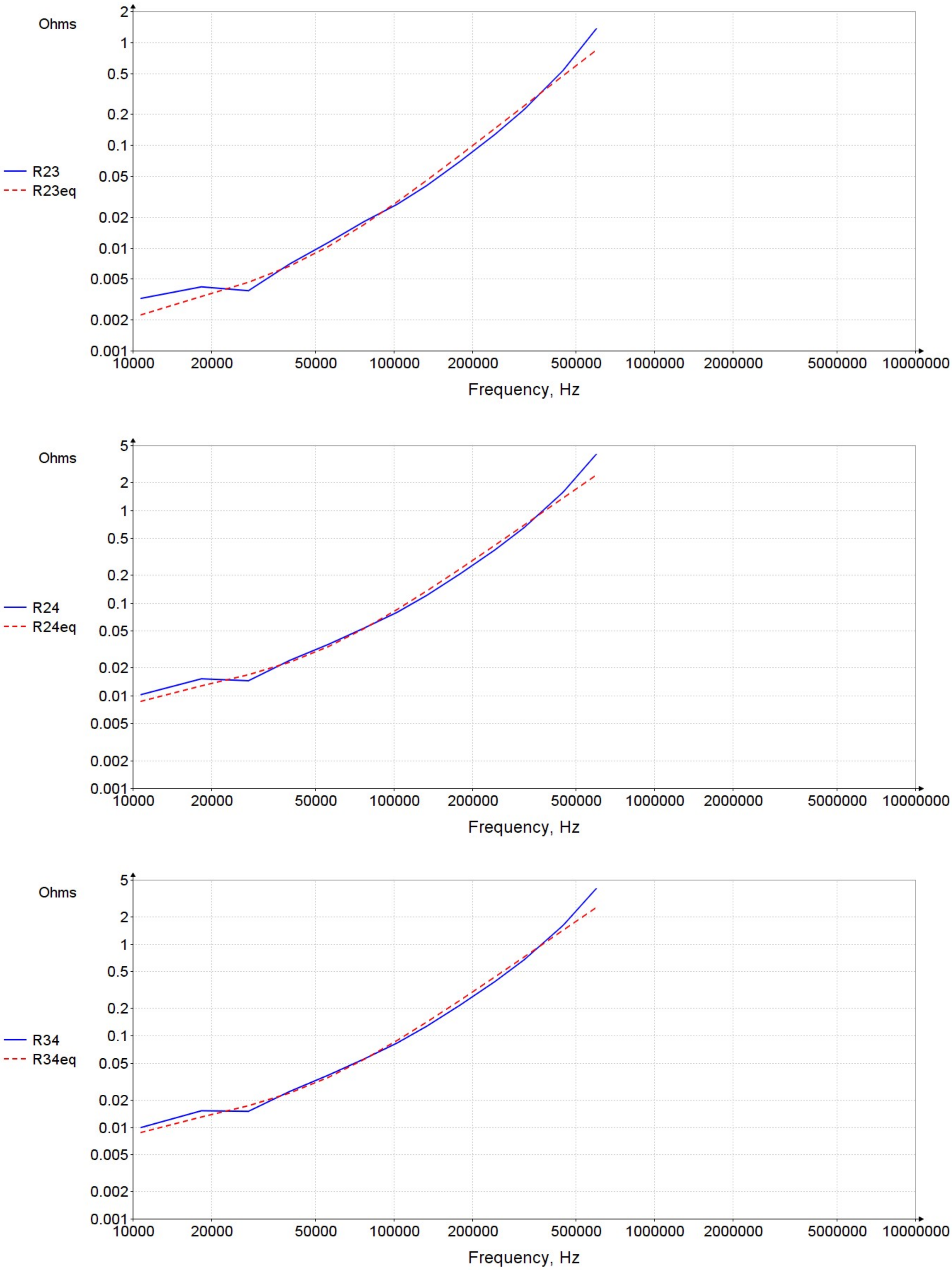


Figure 15b. Measured and Equivalent Circuit mutual resistances.

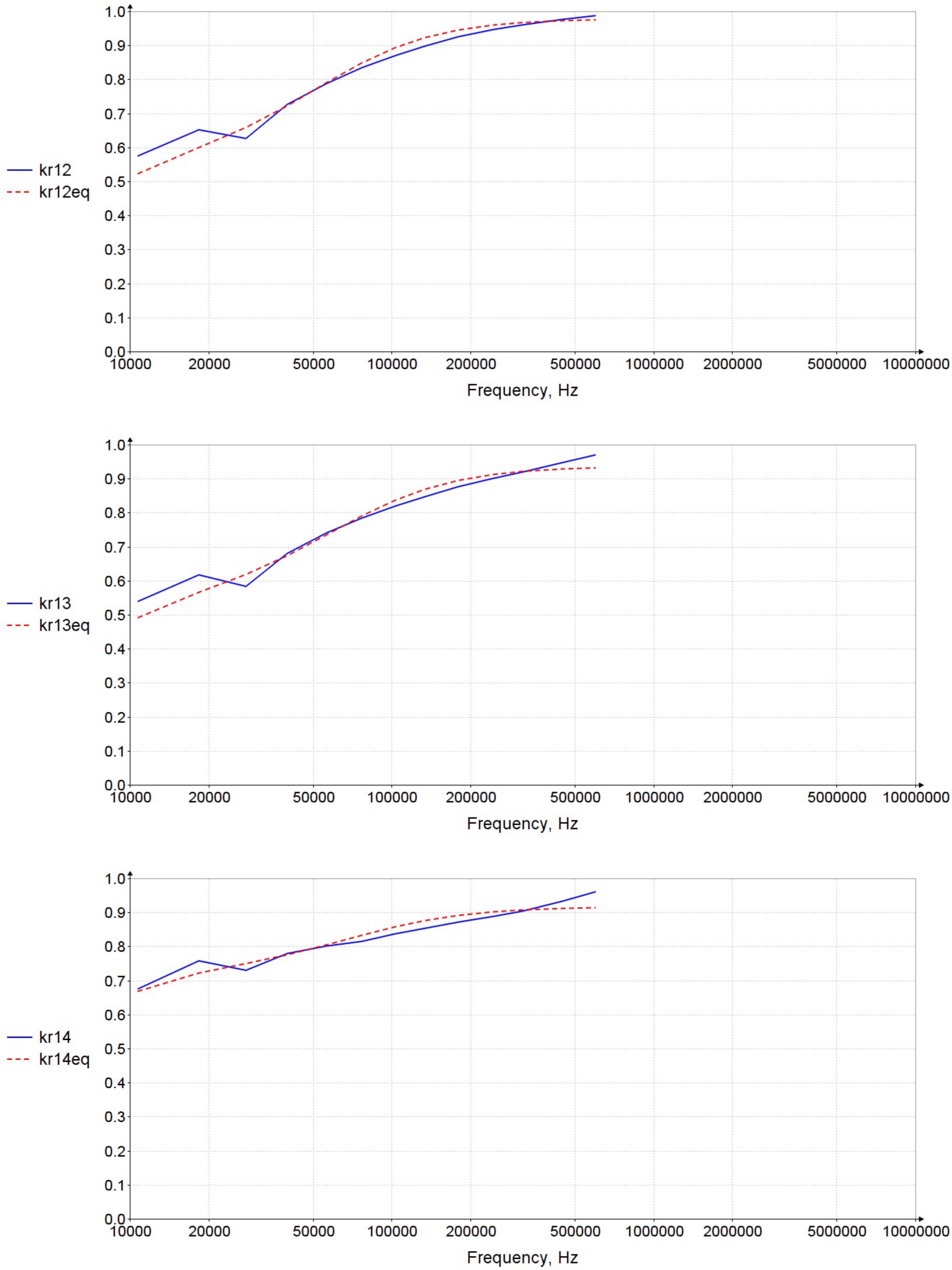


Figure 16a. Measured and Equivalent Circuit mutual resistance couplings.

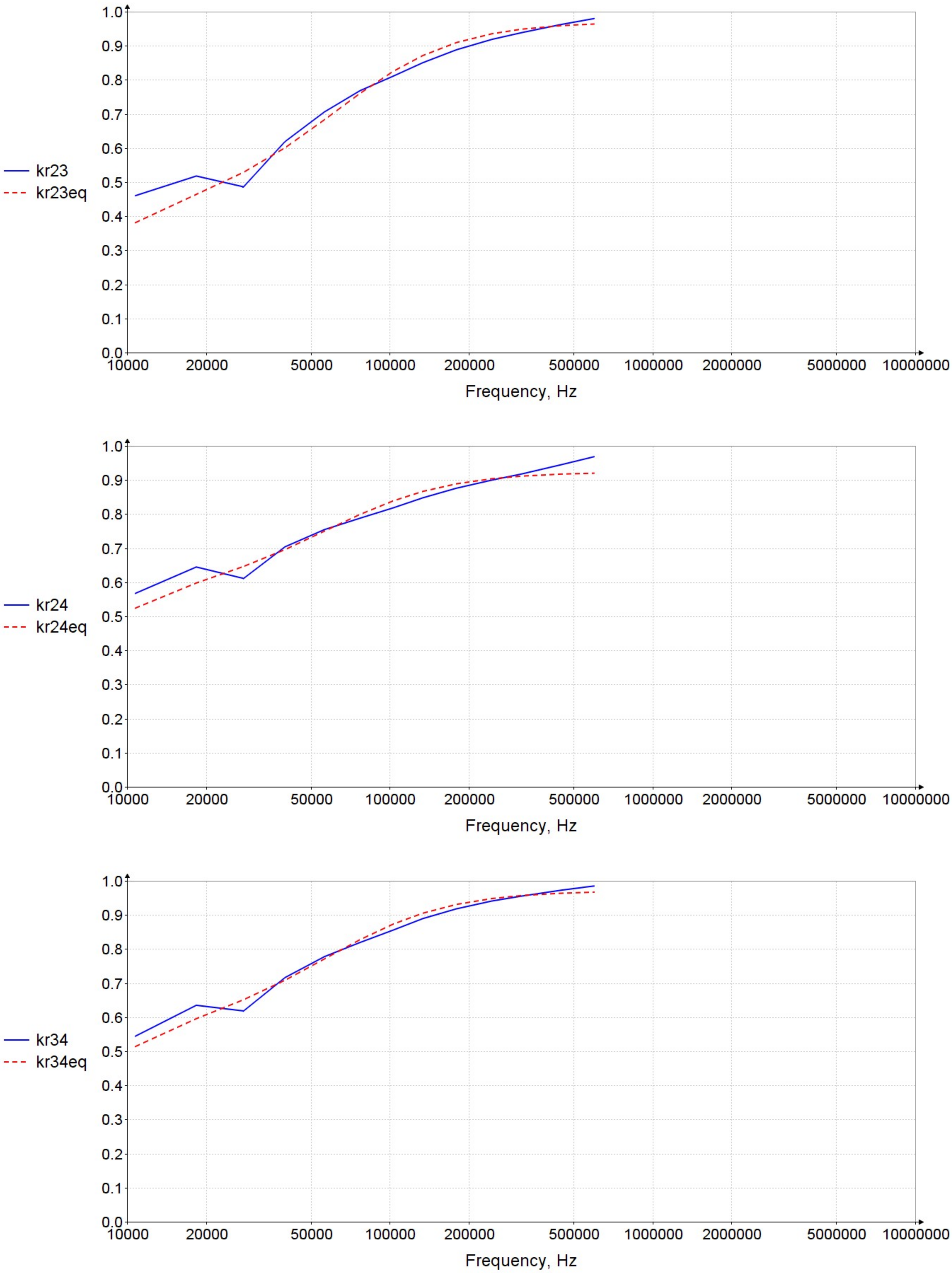


Figure 16b. Measured and Equivalent Circuit mutual resistance couplings.

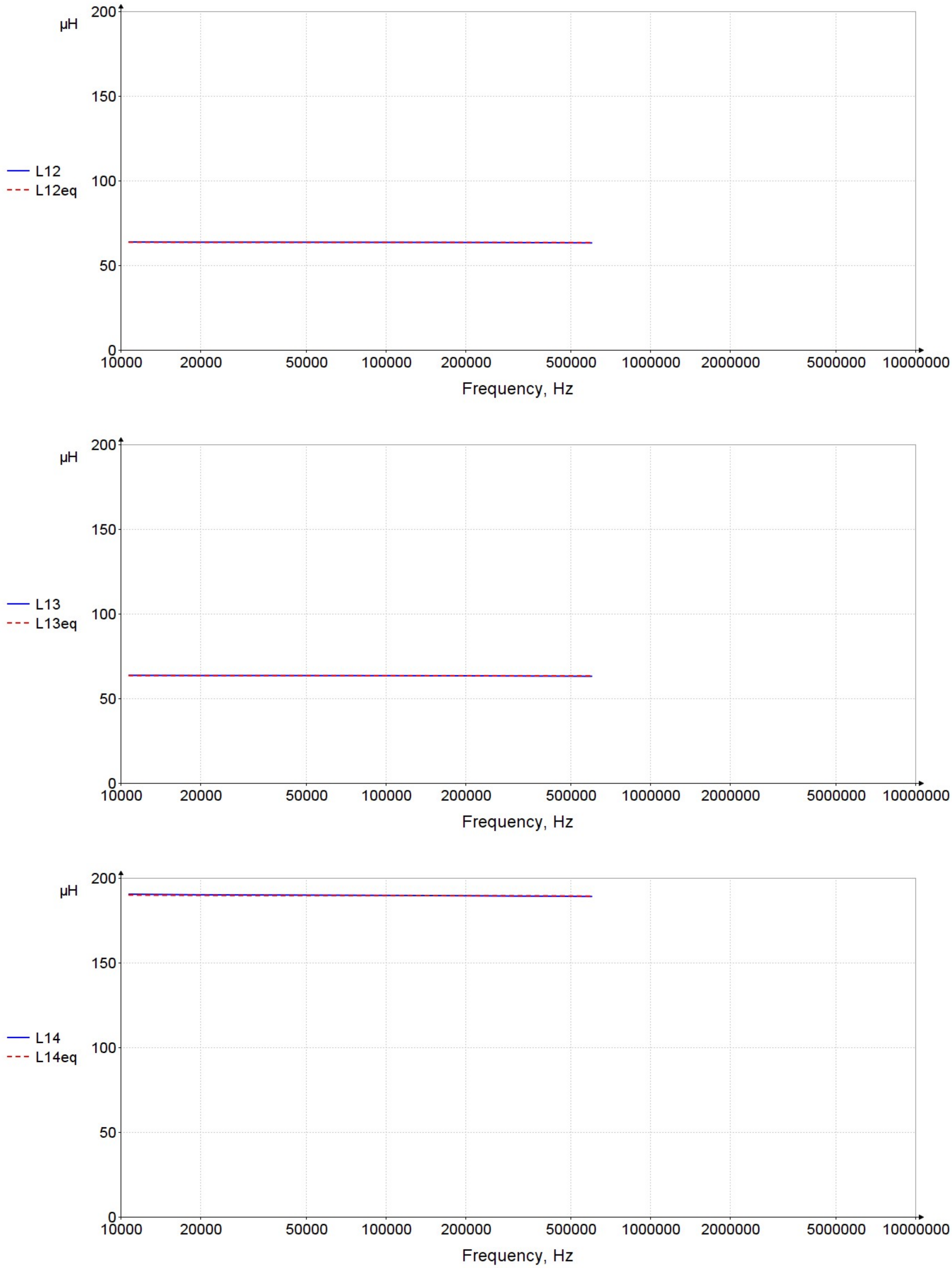
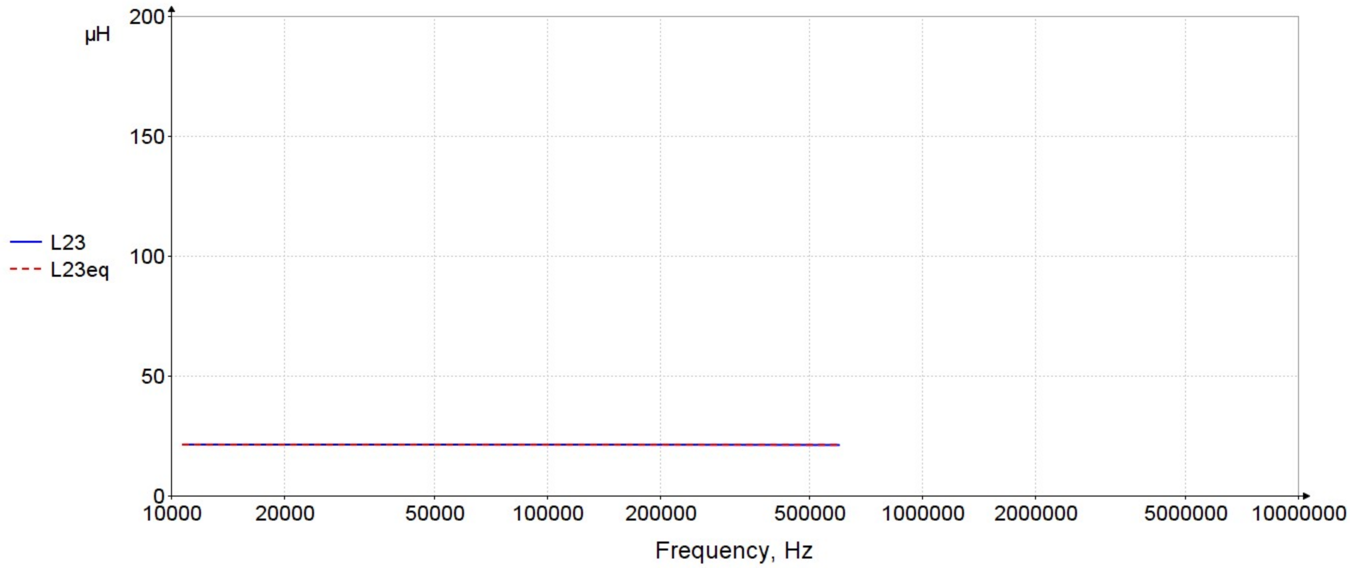
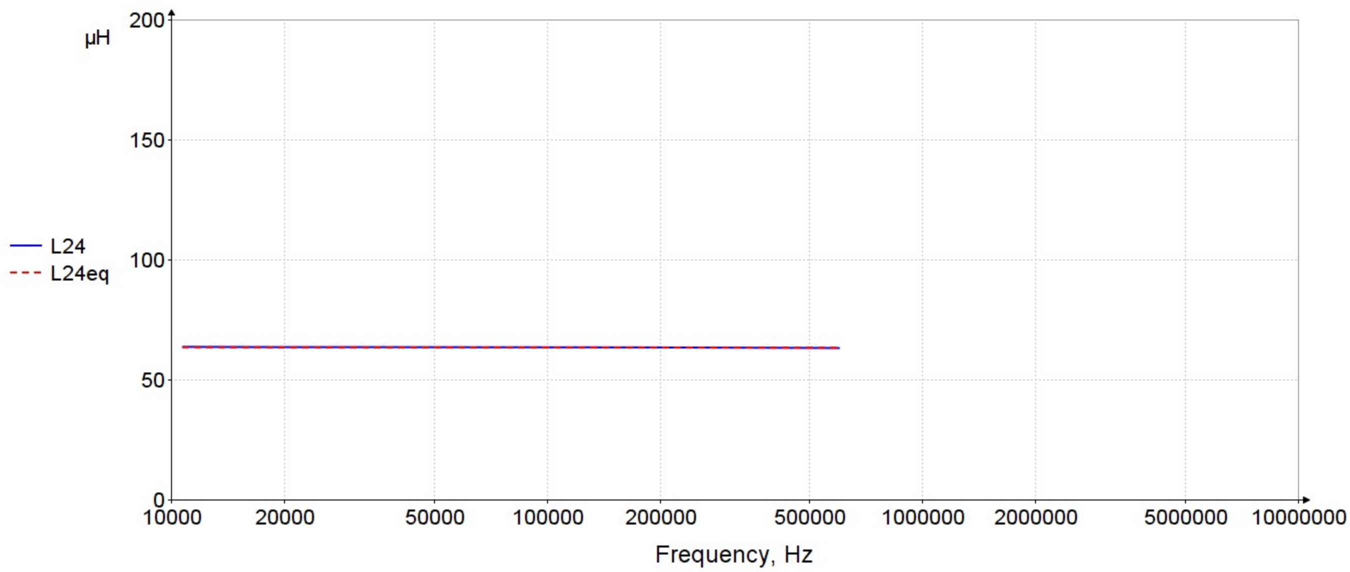


Figure 17a. Measured and Equivalent Circuit mutual inductances.





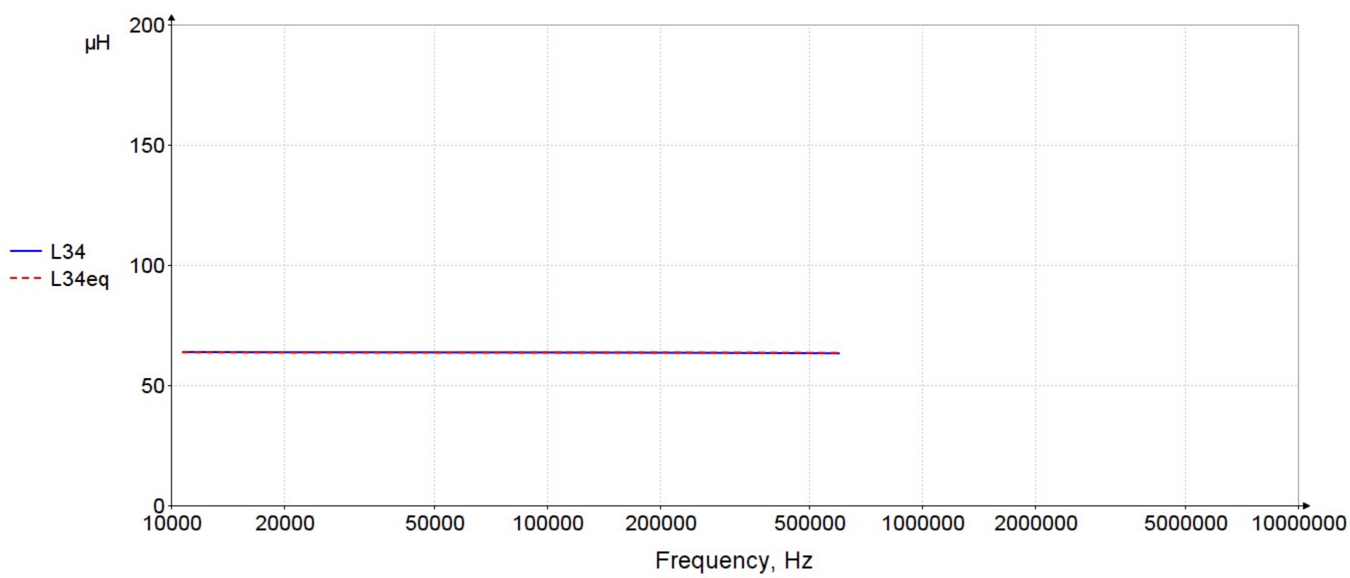


Figure 17b. Measured and Equivalent Circuit mutual inductances.

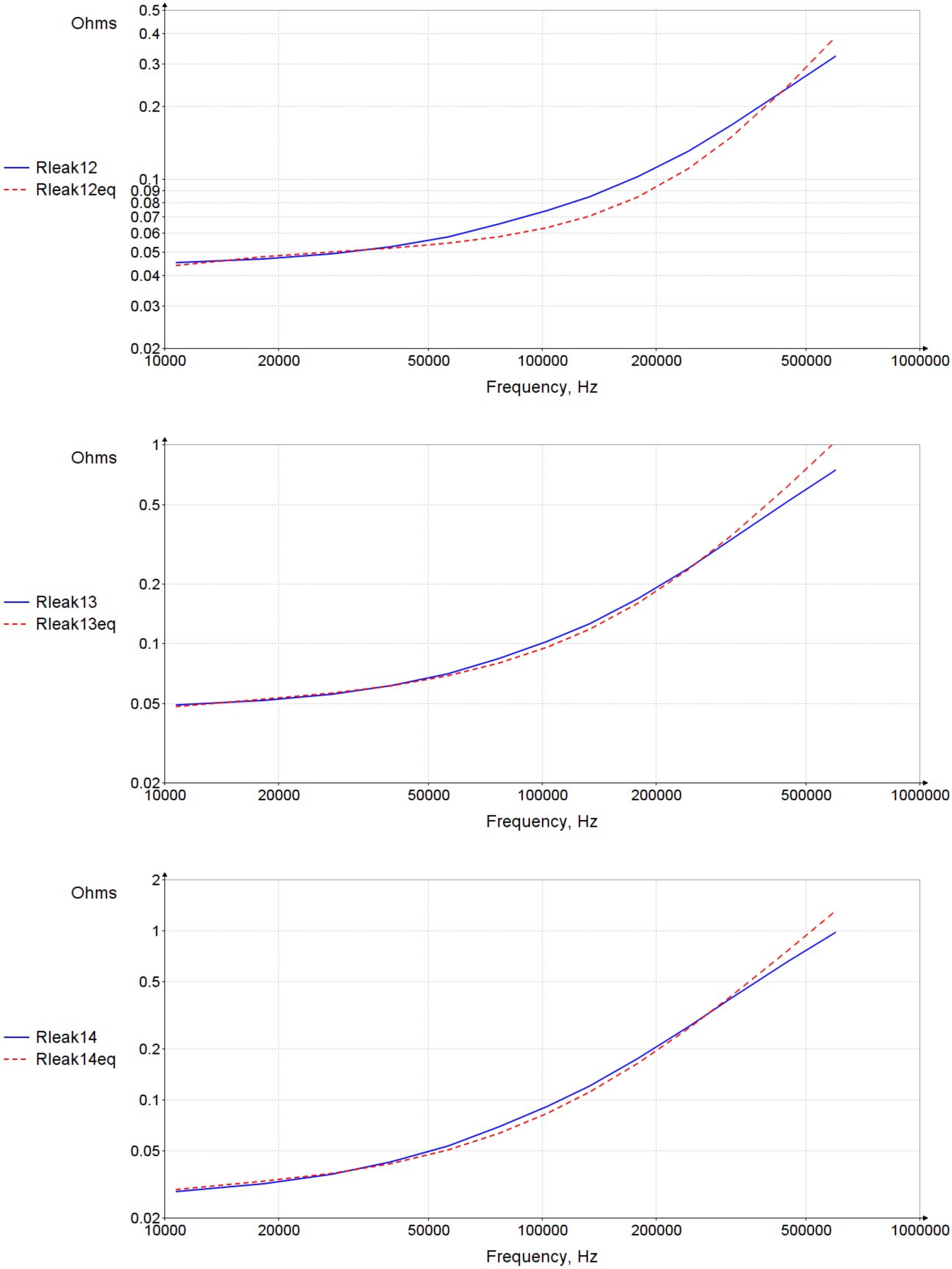
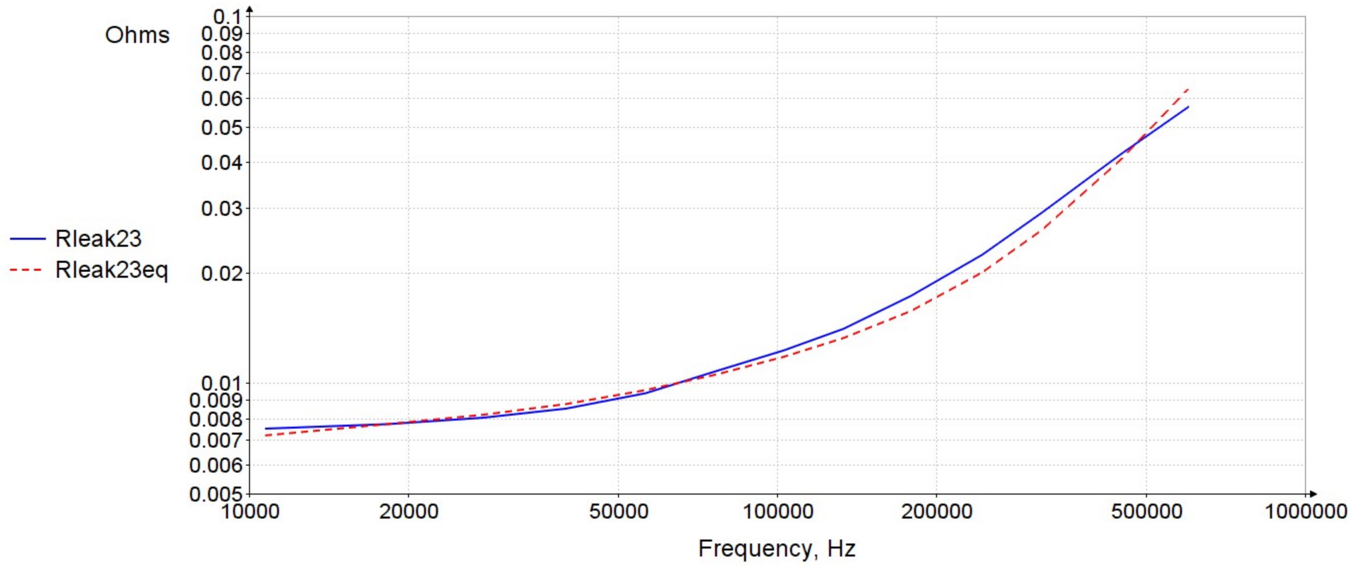
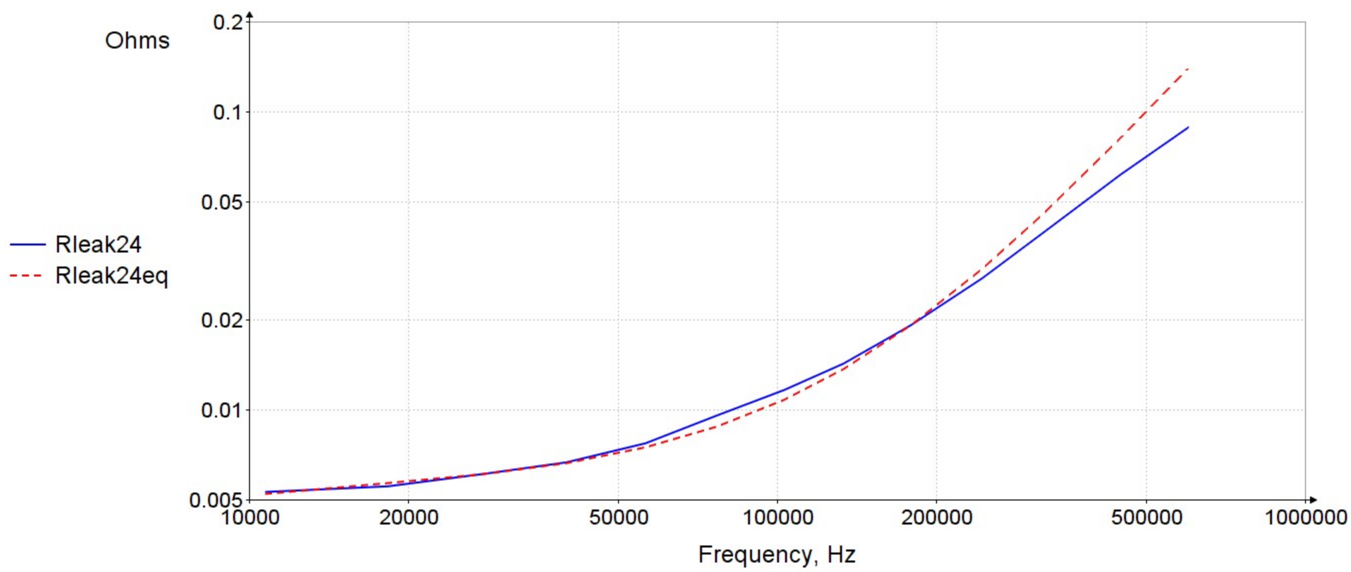


Fig. 18a. Measured and Equivalent Circuit leakage resistances.





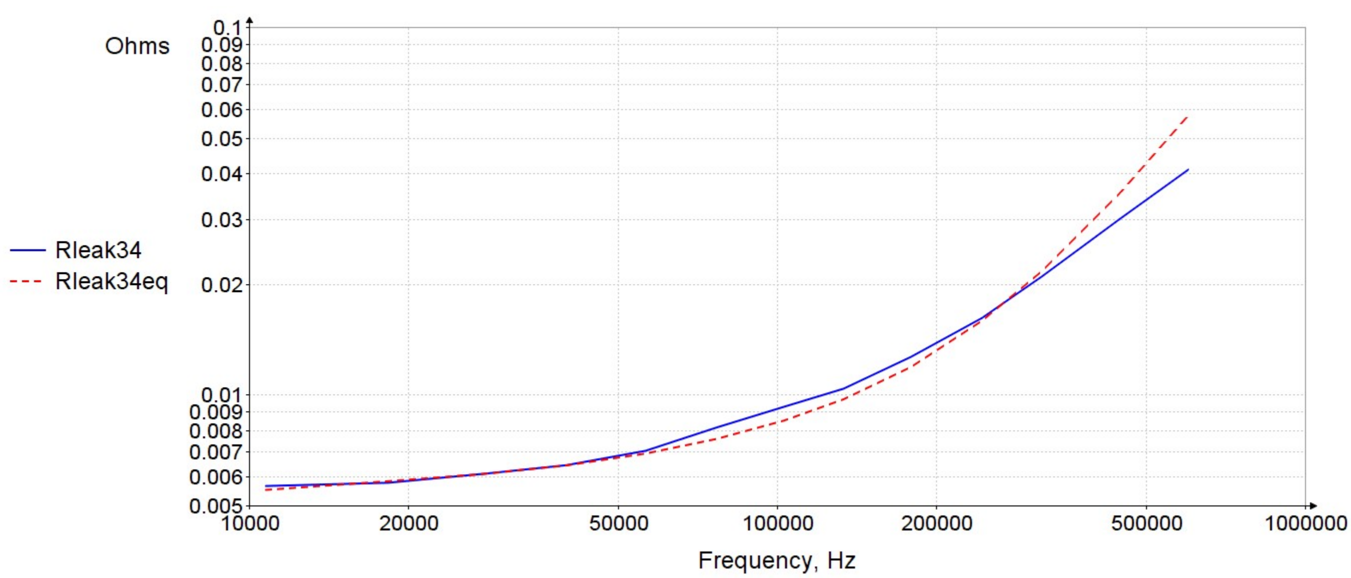


Fig. 18b. Measured and Equivalent Circuit leakage resistances.

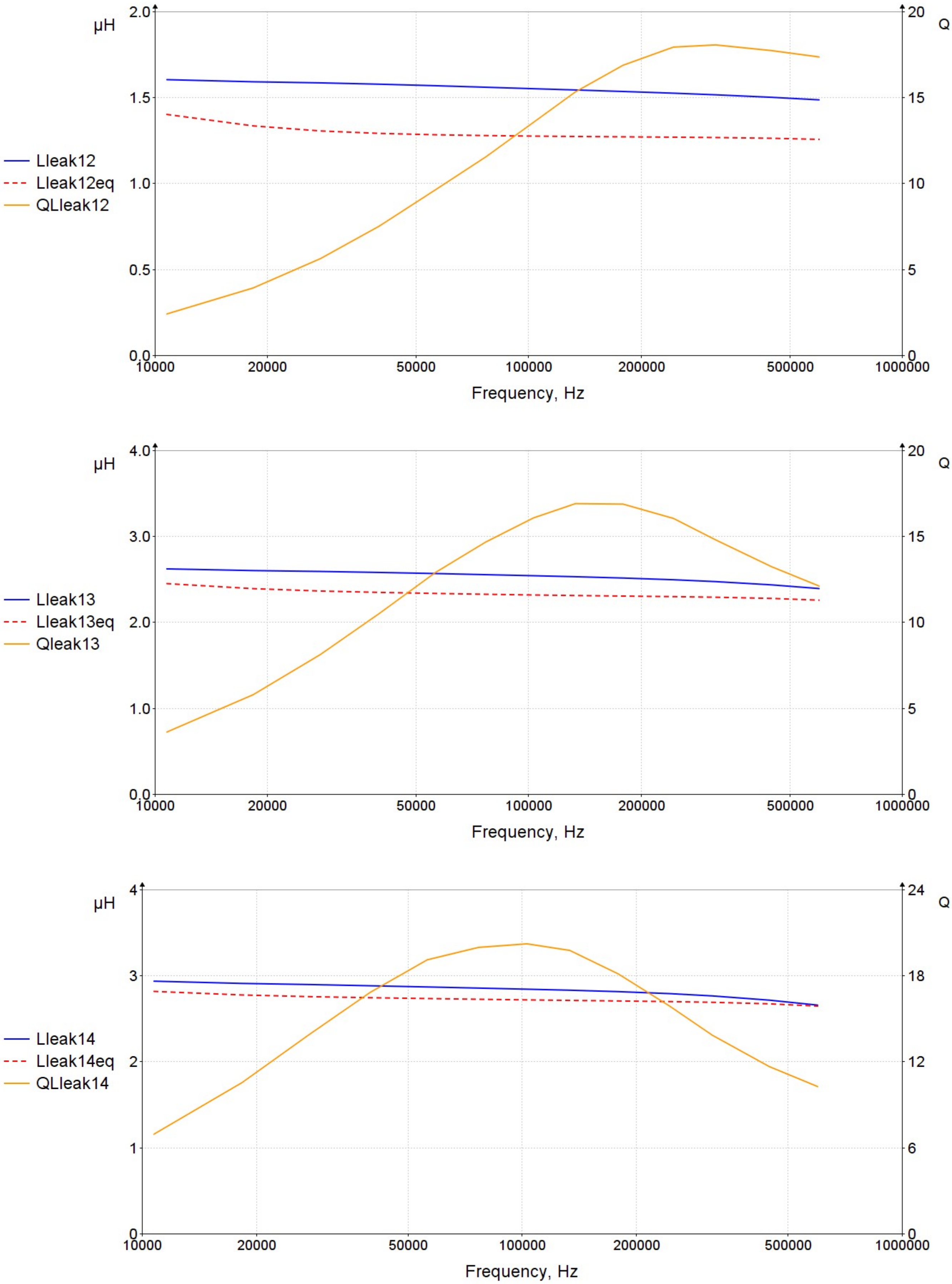
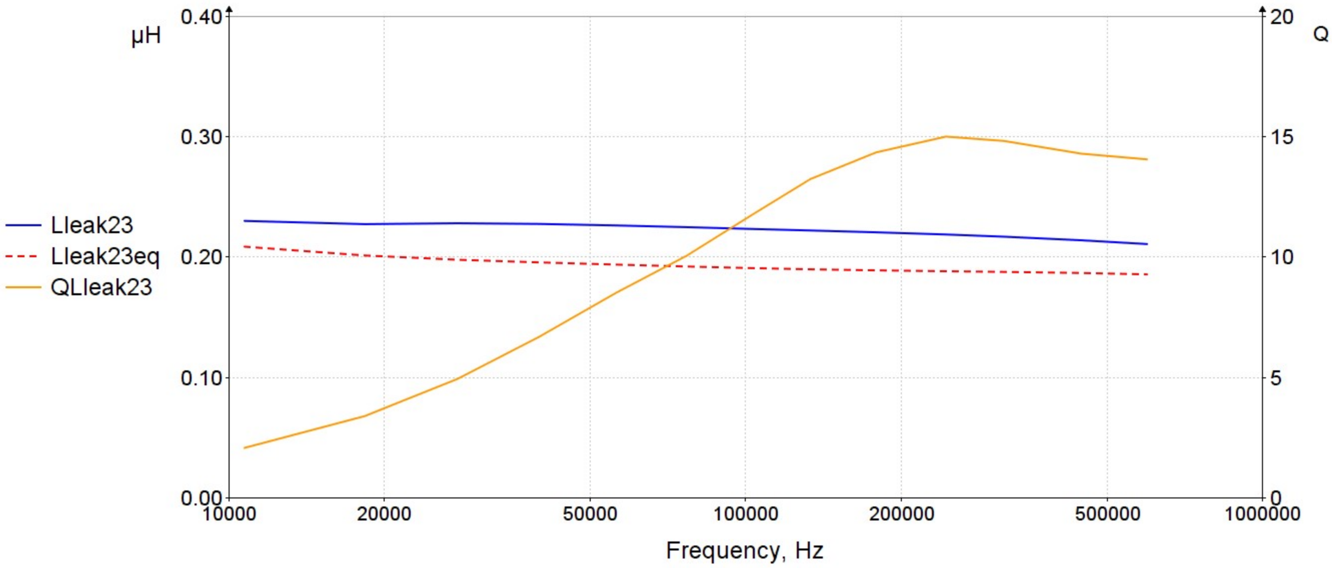
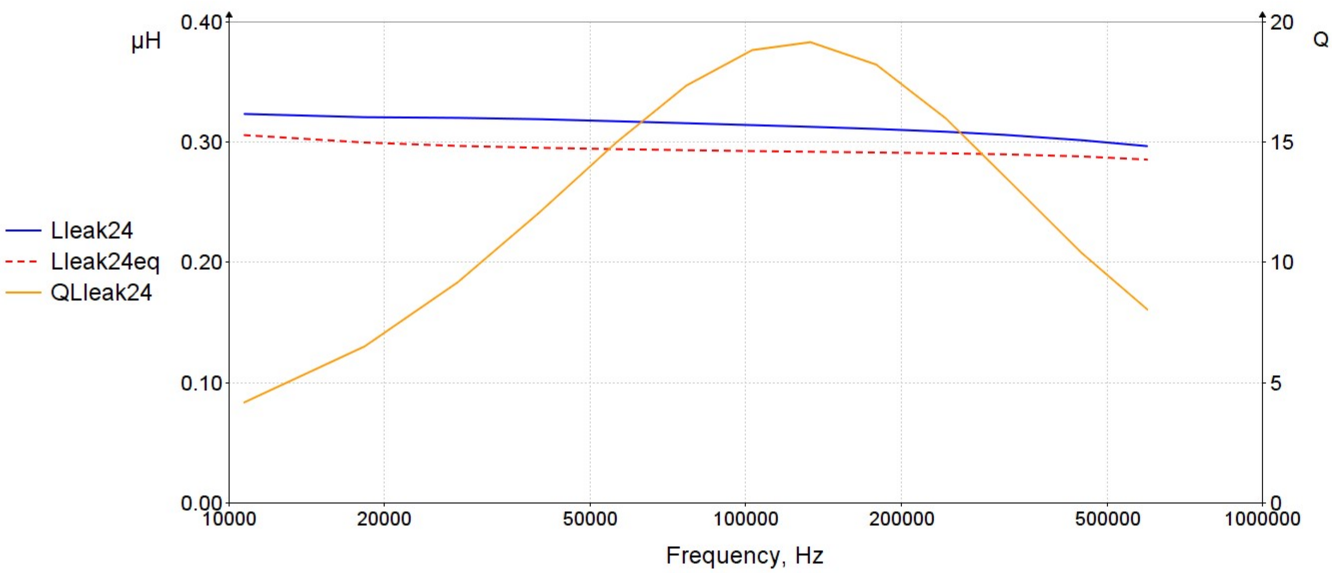


Fig. 19a. Measured and Equivalent Circuit leakage inductances.





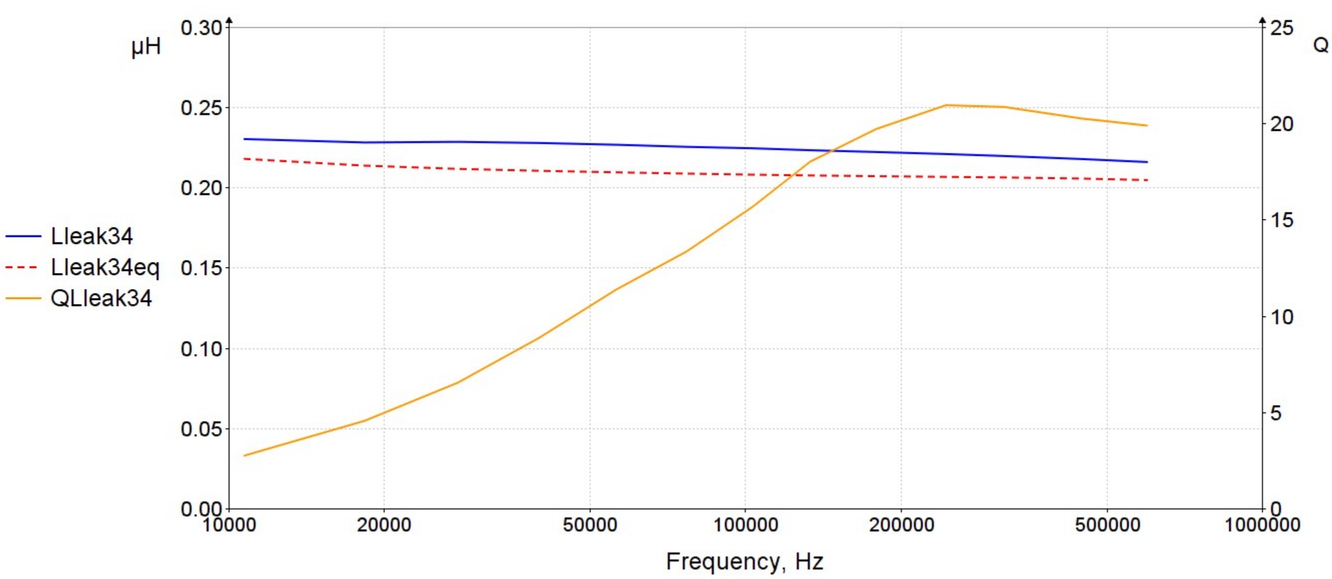


Fig. 19b. Measured and Equivalent Circuit leakage inductances.

References

[1] E. E. Mombello and K. Moller, "New power transformer model for the calculation of electromagnetic resonant transient phenomena including frequency-dependent losses," IEEE Transactions on Power Delivery, vol. 15, No. 1, January 2000, pp. 167-174.
http://ieeexplore.ieee.org/document/847246/

[2] B. L. Hesterman, E. E. Mombello and K. Moller, "Discussion of "New power transformer model for the calculation of electromagnetic resonant transient phenomena including frequency-dependent losses" [Closure to discussion]," IEEE Transactions on Power Delivery, vol. 15, No. no. 4, pp. 1320-1323, Oct. 2000
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[3] Yilmaz Tokad and Myril B. Reed, “Criteria and Tests for Realizability of the Inductance Matrix," Trans. AIEE, Part I, Communications and Electronics, Vol. 78, Jan. 1960, pp. 924-926
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[4] James Spreen, "Electrical terminal representation of conductor loss in transformers," IEEE Transactions on Power Electronics, vol. 5, No. 4, Oct 1990, pp. 424-429.
http://ieeexplore.ieee.org/document/60685/

Calculate strings for exporting the Lb and Rb values to LTspice.

$$paramLB := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{“.param ”} \\ \text{for } m \in 2..\text{rows}(Lb) \\ \quad \left\| \; STR_{m,1} \leftarrow \text{“+ \quad ”} \right. \\ STR \end{array} \right.$$

$$LBnum := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Lb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}(\text{“Lb”}, \text{num2str}(m), \text{“=”}) \right. \\ STR \end{array} \right.$$

$$LB := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Lb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}\left(paramLB_{m}, LBnum_{m}, \text{num2str}\left(\frac{Lb_{m,m}}{\mathbf{H}}\right)\right) \right. \\ STR \end{array} \right.$$

$$paramRB := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{“.param ”} \\ \text{for } m \in 2..\text{rows}(Rb) \\ \quad \left\| \; STR_{m,1} \leftarrow \text{“+ \quad ”} \right. \\ STR \end{array} \right.$$

$$RBnum := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Rb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}(\text{“Rb”}, \text{num2str}(m), \text{“=”}) \right. \\ STR \end{array} \right.$$

$$RB := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(Rb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}\left(paramRB_{m}, RBnum_{m}, \text{num2str}\left(\frac{Rb_{m,m}}{\boldsymbol{\Omega}}\right)\right) \right. \\ STR \end{array} \right.$$

Calculate strings for exporting the RA, LA and KA values.

$$paramRA := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{".param "} \\ \text{for } m \in 2..\text{rows}(R_A) \\ \quad \| \; STR_{m,1} \leftarrow \text{"+ \quad "} \\ STR \end{array} \right.$$

$$RAnum := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } m \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } n \in 1..r \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}(\text{"RA"}, \text{num2str}(m), \text{num2str}(n), \text{"="}) \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ STR \end{array} \right.$$

$$RA := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(R_A) \\ \quad \left\| \; STR_m \leftarrow \text{concat}\left(paramRA_m, RAnum_m, \text{num2str}\left(\frac{R_{A_m}}{\boldsymbol{\Omega}}\right)\right) \right. \\ STR \end{array} \right.$$

$$LAnum := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } \kappa \in 1..r \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}(\text{num2str}(col), \text{num2str}(\kappa)) \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \end{array} \right. \\ STR \end{array} \right.$$

$$KA := \left\| \begin{array}{l} a \leftarrow 1 \\ Cols \leftarrow \text{rows}(L_A) \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1..Cols \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}\left(\text{"KA"}, \text{num2str}(a), \text{" Lb"}, \text{num2str}(row), \text{" LA"}, LAnum_a, \text{" "}, \text{num2str}\left(k_{A_a}\right)\right) \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ STR \end{array} \right.$$

Calculate string for exporting the Kb values.

$$
KB := \left\| \begin{array}{l}
a \leftarrow 1 \\
\text{for } row \in 1..N \\
\quad \left\| \begin{array}{l}
\text{for } col \in row..N \\
\quad \left\| \begin{array}{l}
\text{if } row \neq col \\
\quad \left\| \begin{array}{l}
STR_{a} \leftarrow \text{concat}\left(\text{“Kb”}, \text{num2str}(a), \text{“ Lb”}, \text{num2str}(row), \text{“ Lb”}, \text{num2str}(col), \text{“ ”}, \text{num2str}\left(Kb_{row,col}\right)\right) \\
a \leftarrow a + 1
\end{array} \right.
\end{array} \right.
\end{array} \right. \\
STR
\end{array} \right.
$$

Combine the strings of model parameters for exporting.

$$XFMR\_Params := \text{stack}(LB, RB, RA, KA, KB)$$

Convert each string to its binary representation using str2vec, and add a CR=13 and LF=10 at the end of each string.

$$\textbf{ORIGIN} = 1$$

$$rowCount := \text{rows}(XFMR\_Params) = 74$$

$$indices := \textbf{ORIGIN} .. (rowCount - 1 + \textbf{ORIGIN})$$

$$
XFMR\_Bin := \left\| \begin{array}{l}
resultIndex \leftarrow \textbf{ORIGIN} \\
\text{for } rowIndex \in indices \\
\quad \left\| \begin{array}{l}
row \leftarrow \text{str2vec}\left(XFMR\_Params_{rowIndex}\right) \\
\text{for } colIndex \in \textbf{ORIGIN} .. \text{length}(row) - 1 + \textbf{ORIGIN} \\
\quad \left\| \begin{array}{l}
result_{resultIndex} \leftarrow row_{colIndex} \\
resultIndex \leftarrow resultIndex + 1
\end{array} \right. \\
result_{resultIndex} \leftarrow 13 \\
resultIndex \leftarrow resultIndex + 1 \\
result_{resultIndex} \leftarrow 10 \\
resultIndex \leftarrow resultIndex + 1
\end{array} \right. \\
result
\end{array} \right.
$$

$$\text{WRITEBIN}(\text{“ETD49-25-16\_12-4-4-12T\_Measured\_Rev\_6g.txt”}, \text{“byte”}, 0, XFMR\_Bin) = 0$$