Winding Transformer Model Coefficient Extraction from FEA Data

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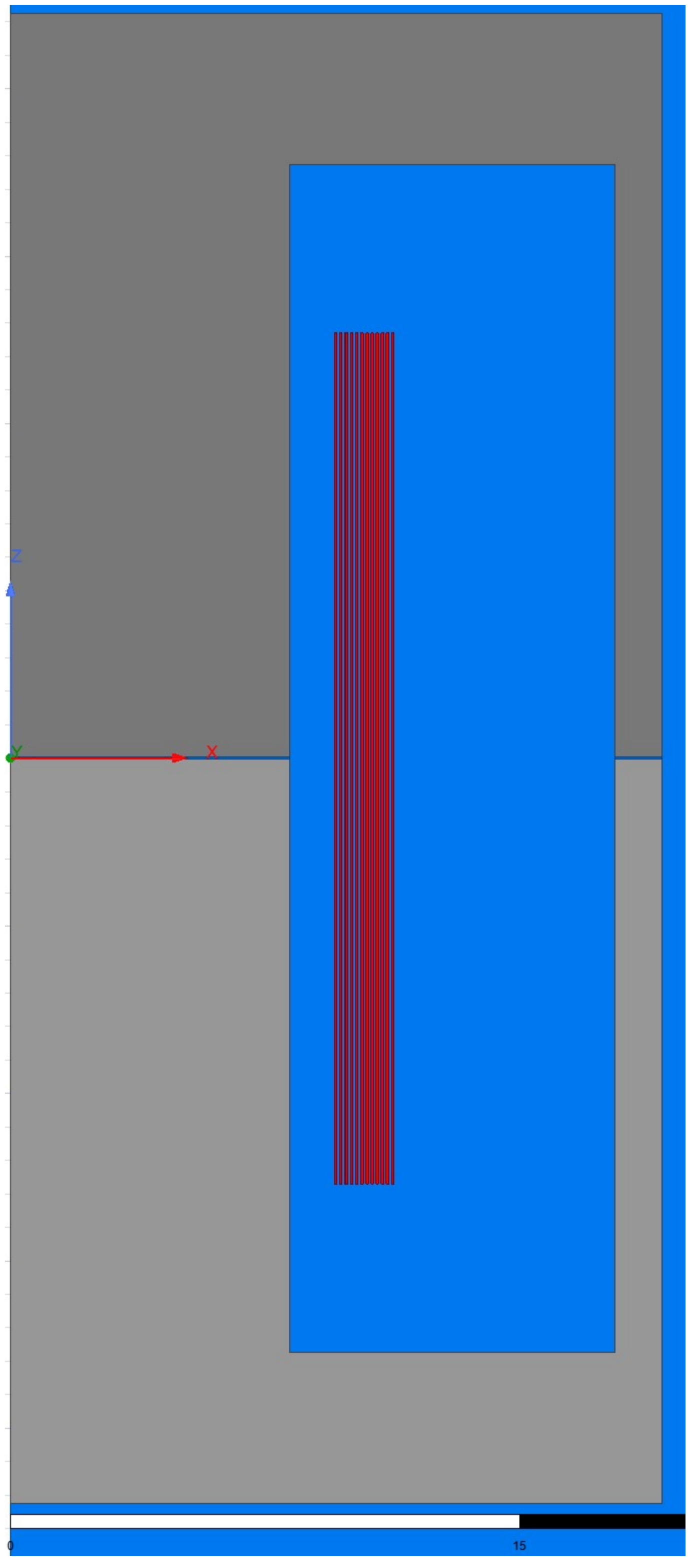
October 10, 2021



Winding 1: 12 T 0.003" by 1" copper foil

(Nomex modeled as 3 mil based on measurements of wound bobbin)

Core: ETD49-25-16 3C97
Gap: 3 mil

Bobbin: Generic ETD49-25-16

Fig. 1. Transformer construction details.

The impedance matrix results of Maxwell 2D simulation ETD49-25-16_12T_10kHz_10MHz.aedt were exported to file: ETD49-25-16_12T_10kHz_10MHz_Setup1.txt.

In order to make parsing for Mathcad easier, that text file was modified and saved as ETD49-25-16_12T_10kHz_10MHz.txt. The following changes were made:

1. The first frequency row starts in the first column like the others.
2. Hz was from the frequencies.
3. Commas were replaced with tabs.
4. Blank rows at the end were deleted.

Read the data file.

$$Maxwell := \mathrm{READFILE}\left(\text{“ETD49-25-16\_12T\_10kHz\_10MHz.txt”}, \text{“delimited”}, 6\right)$$

$$\mathrm{rows}\left(Maxwell\right) = 68 \qquad \mathrm{cols}\left(Maxwell\right) = 5$$

Enter resistance and inductance units used in the data file: $Runit := \boldsymbol{\Omega}$ $\qquad Lunit := 10^{-9} \cdot \boldsymbol{H}$

Determine the number of windings. $Windings := 1$

Determine the number of row per frequency. $RowsPerFreq := Windings + 3 = 4$

Determine the number of frequencies.
(This should be an integer.)

$$N_{freq} := \frac{\mathrm{rows}\left(Maxwell\right)}{RowsPerFreq} - 1 = 16.00$$

Counter variable for the frequency sweep, not including the initial low-frequency simulation. $f := 1 \,..\, N_{freq}$

Extract the frequencies used in the frequency sweep.

$$freq := \begin{array}{|l} \text{for } f \in 1 \,..\, N_{freq} \\ \quad \left| \; freq_f \leftarrow \left(Maxwell\right)_{f \cdot RowsPerFreq + 1,\, 1} \right. \\ freq \cdot \boldsymbol{Hz} \end{array}$$

Separate the data used in the frequency sweep from the initial low-frequency simulation.

$$Sweep := \mathrm{submatrix}\left(Maxwell, RowsPerFreq, \mathrm{rows}\left(Maxwell\right), 4, \mathrm{cols}\left(Maxwell\right)\right)$$

Extract the resistance and inductance data into an array of matrices.

$$RLF := \left\| \begin{array}{l} \text{for } f \in 1..N_{freq} \\ \quad \left\| \begin{array}{l} StartRow \leftarrow 5 + (f-1) \cdot RowsPerFreq \\ StopRow \leftarrow StartRow + Windings - 1 \\ RL_f \leftarrow \text{submatrix}(Sweep, StartRow, StopRow, 1, 2 \cdot Windings) \end{array} \right. \\ RL \end{array} \right.$$

Separate the resistance and inductance data out of RLF into arrays of resistance and inductance matrices.

$$RF := \left\| \begin{array}{l} \text{for } f \in 1..N_{freq} \\ \quad \left\| \begin{array}{l} RL \leftarrow RLF_f \\ \text{for } Row \in 1..Windings \\ \quad \left\| \begin{array}{l} \text{for } Col \in 1..Windings \\ \quad \left\| R_{Row,Col} \leftarrow RL_{Row, 2 \cdot (Col-1)+1} \right. \end{array} \right. \\ RF_f \leftarrow R \end{array} \right. \\ RF \cdot Runit \end{array} \right.$$

$$LF := \left\| \begin{array}{l} \text{for } f \in 1..N_{freq} \\ \quad \left\| \begin{array}{l} RL \leftarrow RLF_f \\ \text{for } Row \in 1..Windings \\ \quad \left\| \begin{array}{l} \text{for } Col \in 1..Windings \\ \quad \left\| L_{Row,Col} \leftarrow RL_{Row, 2 \cdot Col} \right. \end{array} \right. \\ LF_f \leftarrow L \end{array} \right. \\ LF \cdot Lunit \end{array} \right.$$

Create arrays for the resistance and inductance data.

$$R11_f := \left(RF_f\right)_{1,1}$$

$$L11_f := \left(LF_f\right)_{1,1}$$

Counter variables and unit definitions.

$n := 1$ $\quad o := 1$ $\quad m\Omega := \frac{\Omega}{1000}$ $\quad nH := H \cdot 10^{-9}$

A 1 Hz simulation is used to approximate the dc characteristics.

$freq\_min := Maxwell_{1,1} \cdot Hz = 1\ Hz$

Separate the data for the initial low-frequency simulation.

$RL0 := \text{submatrix}(Maxwell, 4, RowsPerFreq, 4, \text{cols}(Maxwell))$

$R_{0_{n,o}} := RL0_{n, 2 \cdot o - 1} \cdot Runit$ $\quad L_{0_{n,o}} := RL0_{n, 2 \cdot o} \cdot Lunit$

The diagonal entries will be used for
the dc values of the winding resistances.

$R_0 = [0.0091]\ \Omega$

The diagonal entries will be used for
the base values of the winding inductances.
The winding losses cause the inductances
to slightly decrease with frequency.

$L_0 = [0.1943]\ mH$

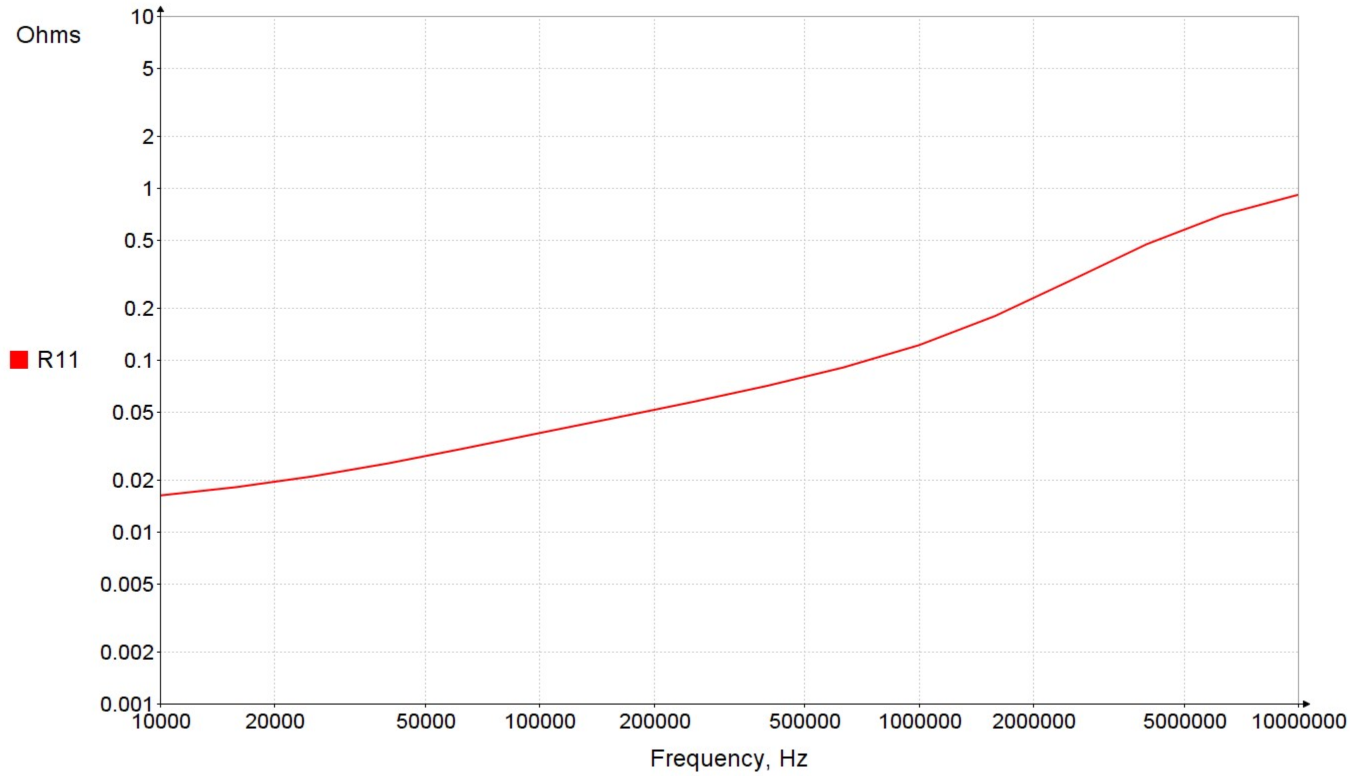


Fig. 2. Simulated self resistances.

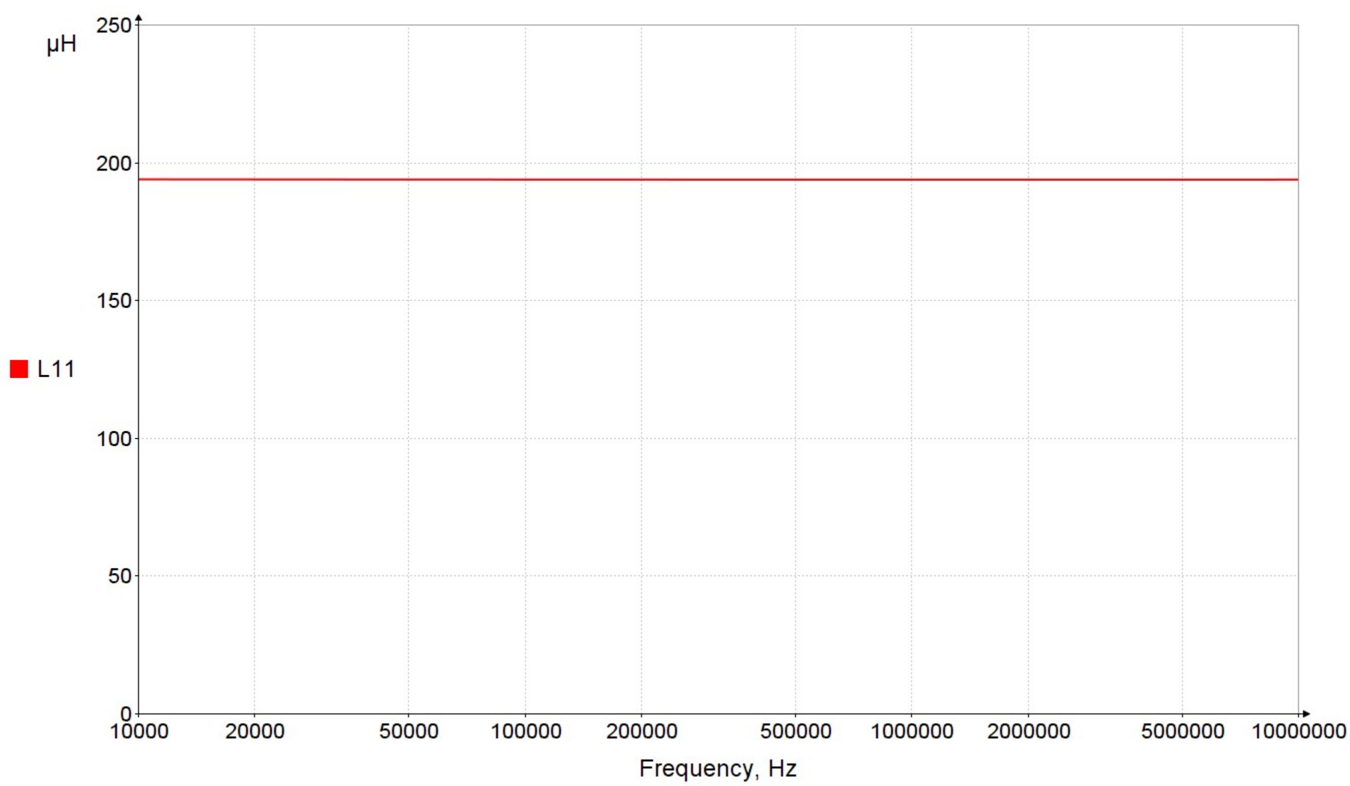


Fig. 3. Simulated self inductances.

The imported simulation data is made available for subsequent calculation through functions that are indexed by the frequency of the matrix data.

$RF(f) := R11_f$ $\qquad$ $LF(f) := L11_f$

Radian frequencies $\qquad \omega_f := 2 \cdot \pi \cdot freq_f$

We begin the process of modeling the transformer by defining the voltages and currents as shown in Fig. 6.

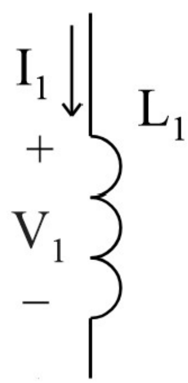


Fig. 6. Voltage and current

The voltage and current can be described in the frequency domain.

$$V_1 = Z_{11} \cdot I_1 \tag{1}$$

The impedance matrix description of the transformer can be approximated with the circuit model shown below in Fig. 12 as explained in [1]. For each winding, there is a resistor representing the dc resistance of that winding and a main inductor representing the maximum low-frequency inductance for that winding. The main inductor and the dc resistance for each winding are connected in series between the electrical terminals of that winding. There is also a set of auxiliary circuits for each winding that is shown in a row to the left of the winding. Each auxiliary circuit consists of an auxiliary inductor that is connected in parallel with an auxiliary resistor. The bottom terminals of all of the inductors in each set are connected together to prevent floating nodes, which are not allowed in circuit simulators.

The schematic diagram shows two auxiliary circuits for each winding, but the model could be extended to include more auxiliary circuits. Increasing the number of auxiliary circuits increases the frequency range over which the skin effect can be modeled.

The main inductors are coupled to each other and to each of the auxiliary inductors. The auxiliary inductors are not coupled to each other. It is, of course, impossible to construct a magnetic device in which a set of uncoupled windings are all coupled some other winding. This arrangement is useful as a model, however, and it is possible to describe it mathematically, and to model it in circuit simulators.

The model has one more degree of freedom than is necessary for each auxiliary inductors, so the inductances of the auxiliary inductors in each set are assigned a value equal to the inductance of the main winding associated with that set.

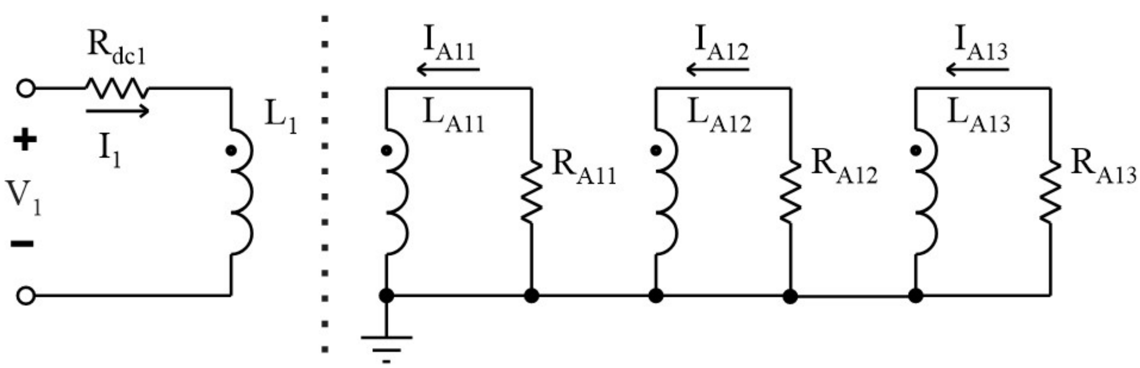


Figure 12. Schematic diagram of inductor circuit model.

We now define several variables and matrices that will be used in an equation that describes the circuit of Fig. 12.

The total number of auxiliary circuits for each winding is designated as r, which is 3 in Fig. 12. The counter variable κ indicates the κth auxiliary circuit, and it ranges from 1 to r.

$N := 1$ $\quad$ $r := 3$ $\quad$ $Naux := N \cdot r = 3$ $\quad$ $Nk := N^2 \cdot r = 3$

The mutual inductance between L_1 and L_2 is designated M_{12}.

The elements of M are calculated as shown below.

$$
\begin{array}{l}
a \leftarrow 1 \\
Cols \leftarrow Naux \\
\text{for } row \in 1..N \\
\quad \text{for } col \in 1..Cols \\
\quad\quad M_{row,col} \leftarrow k_{A_a} \cdot \sqrt{L_{A_{col}} \cdot Lb_{row,row}} \\
\quad\quad a \leftarrow a + 1 \\
M
\end{array}
$$

Define a matrix of the main self and mutual inductances calculated with the initial low-frequency simulation, which was selected to be low enough that the auxiliary circuits have little effect at that frequency.

$$Lb := L_0 \qquad\qquad freq\_min = 1\ Hz$$

$$Lb = \begin{bmatrix} 194.32 \end{bmatrix}\ \mu H$$

Compute the coupling coefficient matrix corresponding to Lb to get an idea of the couplings among the wires. Kb is the coupling coefficient between the main windings.

The value of each auxiliary inductor is set equal to the value of the corresponding main winding.

```
L_A := | row <- 0
       | for n ∈ 1..N
       |   for a ∈ 1..r
       |     row <- row + 1
       |     L_row <- Lb_n,n
       | L
```

R_A contains the initial guess values of the auxiliary resistors for solve block.

k_A contains the initial guess values of the coupling coefficients between the auxiliary circuits and the main windings.

$$R_A := \begin{bmatrix} 10 \\ 100 \\ 1000 \end{bmatrix}\ \Omega$$

```
k_A := | for n ∈ 1..Nk
       |   K_n <- 0.001
       | K
```

$$
M\left(k_A\right) := \left\| \begin{array}{l} a \leftarrow 1 \\ Cols \leftarrow Naux \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1..Cols \\ \quad \left\| \begin{array}{l} M_{row,col} \leftarrow k_{A_a} \cdot \sqrt{L_{A_{col}} \cdot Lb_{row,row}} \\ a \leftarrow a + 1 \end{array} \right. \end{array} \right. \\ M \end{array} \right.
$$

$$Rb := \left[R_{0_{1,1}} \right] \qquad Rb = [9.085]\ m\Omega$$

Functions to define the G and B matrices described in [1]. G and B are needed to find the impedances of the transformer equivalent circuit shown in Fig. 12.

$$
G\left(f, L_A, R_A\right) := \left\| \begin{array}{l} G_A \leftarrow \overrightarrow{\left(\dfrac{R_A}{R_A^{\,2} + \left(\omega_f\right)^2 \cdot L_A^{\,2}} \right)} \\ X \leftarrow \dfrac{\text{identity}(Naux)}{\boldsymbol{\Omega}} \\ \text{for } n \in 1..Naux \\ \quad \left\| X_{n,n} \leftarrow G_{A_n} \right. \\ X \end{array} \right.
$$

$$
B\left(f, L_A, R_A\right) := \left\| \begin{array}{l} B_A \leftarrow \overrightarrow{\left(\dfrac{L_A}{R_A^{\,2} + \left(\omega_f\right)^2 \cdot L_A^{\,2}} \right)} \\ X \leftarrow \text{identity}(Naux) \cdot \dfrac{\boldsymbol{s}}{\boldsymbol{\Omega}} \\ \text{for } n \in 1..Naux \\ \quad \left\| X_{n,n} \leftarrow B_{A_n} \right. \\ X \end{array} \right.
$$

$\text{rows}\left(G\left(1, L_A, R_A\right)\right) = 3$

$\text{cols}\left(G\left(1, L_A, R_A\right)\right) = 3$

$\text{rows}\left(B\left(1, L_A, R_A\right)\right) = 3$

$\text{cols}\left(B\left(1, L_A, R_A\right)\right) = 3$

Function to calculate the equivalent circuit self and mutual resistances.

$$R_{eq}\left(R_A, k_A, f\right) := \left\| \begin{array}{l} R \leftarrow Rb + \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot G\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \left[R_{1,1}\right] \end{array} \right.$$

$$\mathrm{rows}\left(R_{eq}\left(R_A, k_A, 1\right)\right) = 1$$

$$\mathrm{cols}\left(R_{eq}\left(R_A, k_A, 1\right)\right) = 1$$

Function to calculate self and mutual inductances of the the equivalent circuit

$$L_{eq}\left(R_A, k_A, f\right) := \left\| \begin{array}{l} L \leftarrow Lb - \left(\omega_f\right)^2 \cdot M\left(k_A\right) \cdot B\left(f, L_A, R_A\right) \cdot M\left(k_A\right)^{\mathrm{T}} \\ \left[L_{1,1}\right] \end{array} \right.$$

Function to calculate the equivalent circuit self and mutual impedances.

$$Z_{eq}\left(R_A, k_A, f\right) := R_{eq}\left(R_A, k_A, f\right) + 1\mathrm{j} \cdot \omega_f \cdot L_{eq}\left(R_A, k_A, f\right)$$

Define error functions base on resistance and inductance

$$Error\_R\left(R_A, k_A, f\right) := \left\| \begin{array}{l} Rf \leftarrow RF(f) \\ D \leftarrow R_{eq}\left(R_A, k_A, f\right) - Rf \\ \overrightarrow{\dfrac{D}{Rf}} \end{array} \right.$$

$$Error\_L\left(R_A, k_A, f\right) := \left\| \begin{array}{l} Lf \leftarrow LF(f) \\ D \leftarrow L_{eq}\left(R_A, k_A, f\right) - Lf \\ \overrightarrow{\dfrac{D}{Lf}} \end{array} \right.$$

Vectors of zeros used in the Minerr block.

$$Z := \left\| \begin{array}{l} Z_{\frac{N^2+N}{2}} \leftarrow 0 \\ Z \end{array} \right.$$

Error Weighting $\quad c := 10 \quad d := 100$

Guess Values

$d \cdot Error\_R(R_A, k_A, 1) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 5) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 10) = Z$

$d \cdot Error\_R(R_A, k_A, 2) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 6) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 12) = Z$

$d \cdot Error\_R(R_A, k_A, 3) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 7) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 14) = Z$

$d \cdot Error\_R(R_A, k_A, 4) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 8) = Z$ $\quad d \cdot Error\_R(R_A, k_A, 16) = Z$

$c \cdot Error\_L(R_A, k_A, 1) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 5) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 10) = Z$

$c \cdot Error\_L(R_A, k_A, 2) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 6) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 12) = Z$

$c \cdot Error\_L(R_A, k_A, 3) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 7) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 14) = Z$

$c \cdot Error\_L(R_A, k_A, 4) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 8) = Z$ $\quad c \cdot Error\_L(R_A, k_A, 16) = Z$

Constraints

Constraints on coupling coefficients and resistors prevent inappropriate component values.

$-1 < k_{A_1} < 1$

$-1 < k_{A_2} < 1$

$-1 < k_{A_3} < 1$

$10^4 \cdot R_{A_1} > 10^4 \cdot \Omega$

$10^4 \cdot R_{A_2} > 10^4 \cdot \Omega$

$10^4 \cdot R_{A_3} > 10^4 \cdot \Omega$

Solver

$$\begin{bmatrix} R_A \\ k_A \end{bmatrix} := \textbf{Minerr}\left(R_A, k_A\right)$$

$ERR = ?$

$$R_A = \begin{bmatrix} 13.2811 \\ 198.5817 \\ 5.6843 \cdot 10^3 \end{bmatrix} \Omega \qquad k_A = \begin{bmatrix} 0.032 \\ 0.016 \\ 0.013 \end{bmatrix}$$

$$\mathrm{eigenvals}\left(\begin{bmatrix} 1 & k_{A_1} & k_{A_2} \\ k_{A_1} & 1 & k_{A_3} \\ k_{A_2} & k_{A_3} & 1 \end{bmatrix}\right) = \begin{bmatrix} 1.042054 \\ 0.990202 \\ 0.967744 \end{bmatrix}$$

Extract resistance and inductance values of the equivalent circuit for plotting.

$$R11eq_f := R_{eq}\left(R_A, k_A, f\right)_1 \qquad L11eq_f := L_{eq}\left(R_A, k_A, f\right)_1$$

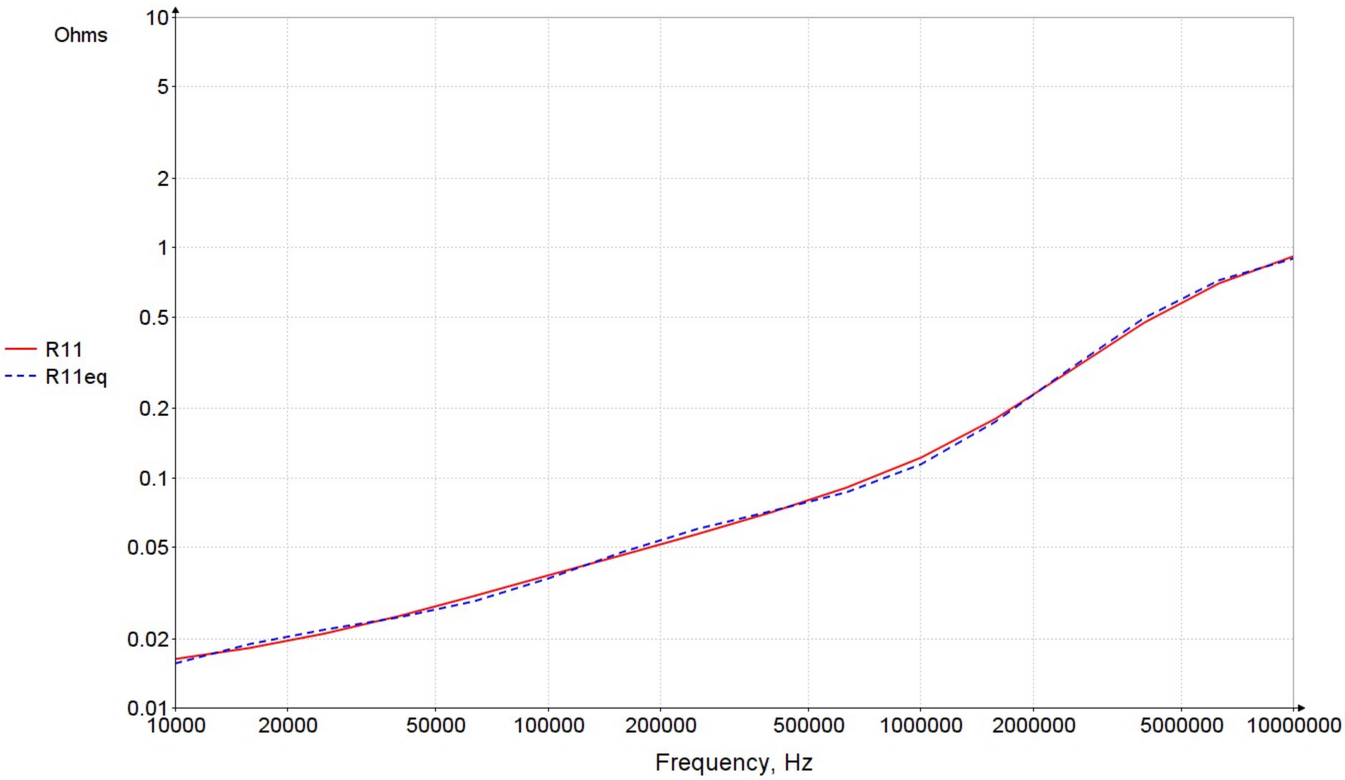


Figure 13. FEA and Equivalent Circuit self resistances.

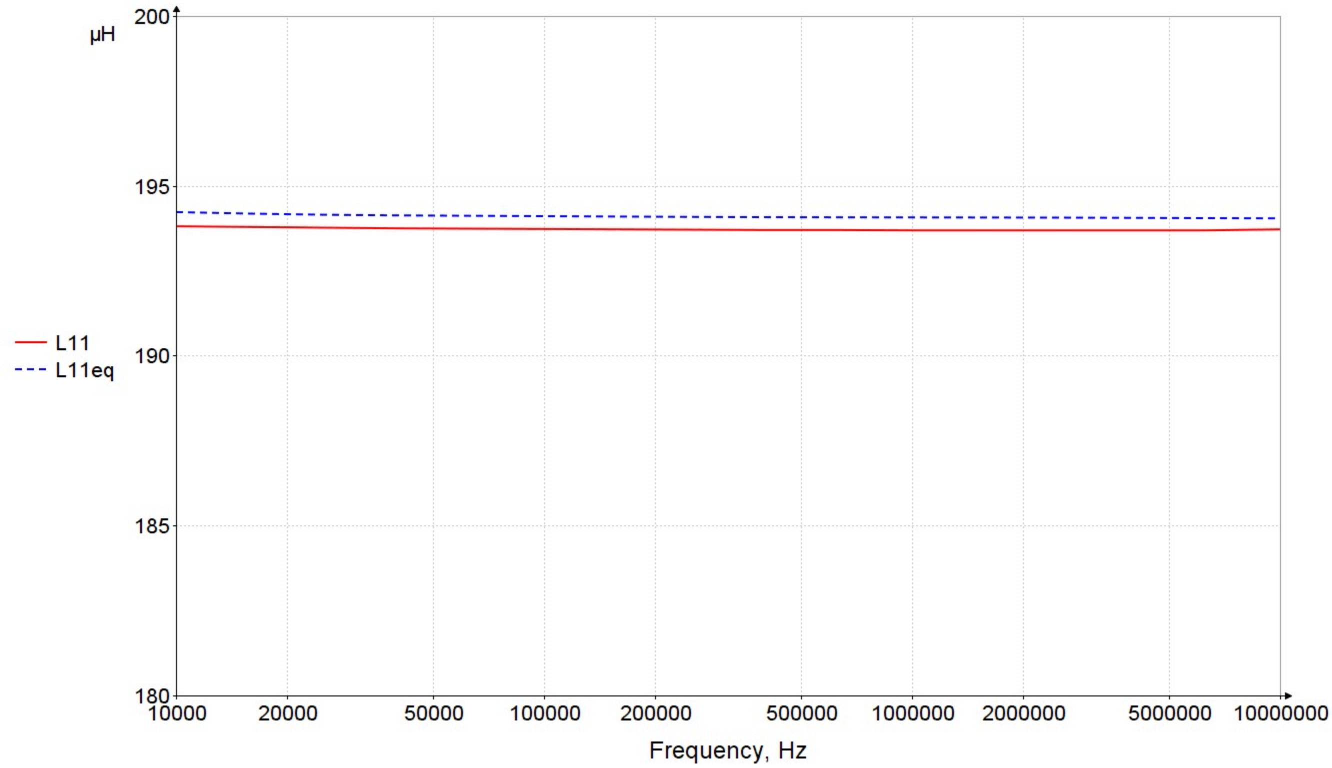


Figure 14. FEA and Equivalent Circuit self inductances.

References

[1] E. E. Mombello and K. Moller, "New power transformer model for the calculation of electromagnetic resonant transient phenomena including frequency-dependent losses," IEEE Transactions on Power Delivery, vol. 15, No. 1, January 2000, pp. 167-174. http://ieeexplore.ieee.org/document/847246/

[2] B. L. Hesterman, E. E. Mombello and K. Moller, "Discussion of "New power transformer model for the calculation of electromagnetic resonant transient phenomena including frequency-dependent losses" [Closure to discussion]," IEEE Transactions on Power Delivery, vol. 15, No. no. 4, pp.

losses" [Closure to discussion]," IEEE Transactions on Power Delivery, vol. 15, No. no. 4, pp. 1320-1323, Oct. 2000
http://ieeexplore.ieee.org/document/847246/

[3] Yilmaz Tokad and Myril B. Reed, "Criteria and Tests for Realizability of the Inductance Matrix," Trans. AIEE, Part I, Communications and Electronics, Vol. 78, Jan. 1960, pp. 924-926
http://ieeexplore.ieee.org/document/6368492/

[4] James Spreen, "Electrical terminal representation of conductor loss in transformers," IEEE Transactions on Power Electronics, vol. 5, No. 4, Oct 1990, pp. 424-429.
http://ieeexplore.ieee.org/document/60685/

Calculate strings for exporting the Lb and Rb values to LTspice.

$$paramLB := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{“.param ”} \\ STR \end{array} \right.$$

$$LBnum := \left\| \begin{array}{l} \text{for } m \in 1\,..\,\text{rows}(Lb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}\left(\text{“Lb”}, \text{num2str}(m), \text{“=”}\right) \right. \\ STR \end{array} \right.$$

$$LB := \left\| \begin{array}{l} \text{for } m \in 1\,..\,\text{rows}(Lb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}\left(paramLB_{m}, LBnum_{m}, \text{num2str}\left(\frac{Lb_{m,m}}{\boldsymbol{H}}\right)\right) \right. \\ STR \end{array} \right.$$

$$paramRB := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{“.param ”} \\ \text{for } m \in 2\,..\,\text{rows}(Rb) \\ \quad \left\| \; STR_{m,1} \leftarrow \text{“+ \quad ”} \right. \\ STR \end{array} \right.$$

$$RBnum := \left\| \begin{array}{l} \text{for } m \in 1\,..\,\text{rows}(Rb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}\left(\text{“Rb”}, \text{num2str}(m), \text{“=”}\right) \right. \\ STR \end{array} \right.$$

$$RB := \left\| \begin{array}{l} \text{for } m \in 1\,..\,\text{rows}(Rb) \\ \quad \left\| \; STR_{m} \leftarrow \text{concat}\left(paramRB_{m}, RBnum_{m}, \text{num2str}\left(\frac{Rb_{m,m}}{\boldsymbol{\Omega}}\right)\right) \right. \\ STR \end{array} \right.$$

Calculate strings for exporting the RA, LA and KA values.

$$paramRA := \left\| \begin{array}{l} STR_{1,1} \leftarrow \text{".param "} \\ \text{for } m \in 2..\text{rows}(R_A) \\ \quad \left\| STR_{m,1} \leftarrow \text{"+ \quad "} \right. \\ STR \end{array} \right.$$

$$RAnum := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } m \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } n \in 1..r \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}(\text{"RA"}, \text{num2str}(m), \text{num2str}(n), \text{"="}) \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ STR \end{array} \right.$$

$$RA := \left\| \begin{array}{l} \text{for } m \in 1..\text{rows}(R_A) \\ \quad \left\| STR_m \leftarrow \text{concat}\left(paramRA_m, RAnum_m, \text{num2str}\left(\frac{R_{A_m}}{\boldsymbol{\Omega}}\right)\right) \right. \\ STR \end{array} \right.$$

$$LAnum := \left\| \begin{array}{l} a \leftarrow 1 \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } \kappa \in 1..r \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}(\text{num2str}(col), \text{num2str}(\kappa)) \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \end{array} \right. \\ STR \end{array} \right.$$

$$KA := \left\| \begin{array}{l} a \leftarrow 1 \\ Cols \leftarrow \text{rows}(L_A) \\ \text{for } row \in 1..N \\ \quad \left\| \begin{array}{l} \text{for } col \in 1..Cols \\ \quad \left\| \begin{array}{l} STR_a \leftarrow \text{concat}\left(\text{"KA"}, \text{num2str}(a), \text{" Lb"}, \text{num2str}(row), \text{" LA"}, LAnum_a, \text{" "}, \text{num2str}\left(k_{A_a}\right)\right) \\ a \leftarrow a+1 \end{array} \right. \end{array} \right. \\ STR \end{array} \right.$$

Combine the strings of model parameters for exporting.

$XFMR\_Params := \mathrm{stack}\left(LB, RB, RA, KA\right)$

Convert each string to it's binary representation using str2vec, and add a CR=13 and LF=10 at the end of each string.

$\mathbf{ORIGIN} = 1$

$rowCount := \mathrm{rows}\left(XFMR\_Params\right) = 8$

$indices := \mathbf{ORIGIN} .. \left(rowCount - 1 + \mathbf{ORIGIN}\right)$

```
XFMR_Bin := | resultIndex ← ORIGIN
            | for rowIndex ∈ indices
            |   | row ← str2vec(XFMR_Params_rowIndex)
            |   | for colIndex ∈ ORIGIN .. length(row) − 1 + ORIGIN
            |   |   | result_resultIndex ← row_colIndex
            |   |   | resultIndex ← resultIndex + 1
            |   | result_resultIndex ← 13
            |   | resultIndex ← resultIndex + 1
            |   | result_resultIndex ← 10
            |   | resultIndex ← resultIndex + 1
            | result
```

WRITEBIN("ETD49-25-16_12T.txt", "byte", 0, $XFMR\_Bin$) = 0